Phonon assisted two-qubit subradiant state preparation

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Abstract — **We investigate phonon asisted two-qubit subradiant state population in the long time limit. The system of interest consists from a laser-pumped two-level qubit pair embedded in a substrate. The subradiant state gets populated in the steady-state and this occurs due to phonon processes mediated by the laser pumping effects.**

Key words — **two-level qubits, subradiant states, entanglement, phonons.**

I. INTRODUCTIONS

Quantum dot samples are one of the most studied quantum optical systems at the moment. They are subject of great interest in engineering due to its unique properties. Behaving to a certain degree like real atoms, quantum dot samples may play a crucial role in future quantum technologies. In recent years, there has been substantial progress in this area, with many papers being published. Particularly, the collective interaction of excitons in closely spaced artificial molecules and arrays of nearly identical quantum dots with the electromagnetic modes were excessively researched in [1], including the spontaneous emission of phonons by coupled quantum dots [2]. Fast phonon dynamics of a nanomechanical oscillator due to cooperative effects was theoretically investigated [3].

In the present paper, we analyze the time evolution of initially laser-pumped and ground-state qubits with the aim to populate the subradiant two-particle state of the system. We have found that the phonon subsystem leads to subradiant two-qubit state population due to an additionally phonon induced decay channel.

II. THE MODEL AND ANALYTICAL FORMALISM

We consider a laser pumped system consisting of a pair of identical two-level quantum dots and the laser wave-vector is perpendicular to the line connecting the qubits. The two-level qubits interact with the environmental electromagnetic field reservoir, phonon thermostat as well as with an external coherent laser source. The master equation describing the investigated model in the Born-Markov approximations can be represented as follows [4-6]:

$$\dot{\rho} + \frac{i}{\hbar}[H,\rho] = -\frac{\gamma}{2}(1+\chi_r)\left[R_{es}+R_{sg},\left(R_{se}+R_{gs}\right)\rho\right]$$

$$-\frac{\gamma}{2}(1-\chi_r)\left[R_{ea}-R_{ag},\left(R_{ae}-R_{ga}\right)\rho\right]$$

$$-\Gamma(1+\bar{n})[R_{sa}, R_{as}\rho] - \Gamma\bar{n}[R_{as}, R_{sa}\rho] + \text{H.c} \qquad (1)$$

here

$$\Gamma = \frac{\pi}{4}\sum_{\xi=\{1,2\}}\sum_{p\xi}\left(\lambda_{p\xi}^{(\xi)}/\hbar\right)^2\delta(\omega_{p\xi}-2\Omega_{dd}) \qquad (2)$$

where $\Omega_{dd}>0$,is the decay rate among the symmetrical and antisymmetrical two-qubit states due to phonon thermostat.

In Eq.(1), the Hamiltonian characterizing the coherent quantum dynamics of the qubits interacting with the laser field is

$$H = H_o + H_i, \qquad (3)$$

where

$$\frac{H_0}{\hbar} = 2\Delta R_{ee} + (\Delta+\Omega_{dd})R_{ss} + (\Delta-\Omega_{dd})R_{aa} \qquad (4)$$

and

$$H_i = \sqrt{2}\hbar\Omega(R_{es}+R_{sg}+R_{se}+R_{gs}) \qquad (5)$$

Here, Ω is the corresponding Rabi frequency, Δ is the detuning of the two-level qubit's frequency from the laser one. χ_r the signify tradiative coupling amongthe two-level qubits while Ω_{dd} corresponds to the dipole-dipole interaction potential, respectively. The radiative coupling χ_r goes to zero (unity) for larger (smaller) interparticle separations *r* in comparison to the photon emission wavelength. Corres-pondingly, Ω_{dd} tends to zero or to the static dipole-dipole interaction potential,namely, $\Omega_{dd} = \frac{3\gamma(1-3cos^2\zeta)}{4(kr)^3}$ where ζ is the ngle between the transition dipole vector $\vec{d}$ and the vector connecting the two qubits, i.e., $\vec{r}$. The single-qubit spontaneous decay rate is $\gamma = \frac{k^3d^2}{6\pi\epsilon\epsilon_0\hbar}$, where *k* is the resonant photon wave-vector in the medium, while ϵ_0 is the vacuum permitivity whereas ϵ is the relative dielectric constant of the semiconductor. The two-qubit transition operators are obtained using the common Dicke states, namely, $R_{\alpha\beta} = |\alpha\rangle\langle\beta|$, where $\{\alpha, \beta\} \in$ {e, g, s, a} denote the two-qubit excited state and the ground state, and the symmetrical and antisymmetrical

collective states, respectively. They obey the commutation relation: $[R_{\alpha\beta}, R_{\alpha'\beta'}] = \delta_{\beta\beta'} R_{\alpha\alpha'} - \delta_{\alpha'\alpha} R_{\beta'\beta}$ Notice that the collective two-qubit states are defined as follows [4,5].
Where we have used the two-qubit collective states (see Fig.1)

$$
\begin{aligned}
|e\rangle &= |e_a\rangle\,|e_b\rangle \\
|g\rangle &= |g_a\rangle\,|g_b\rangle \qquad (6) \\
|s\rangle &= \frac{1}{\sqrt{2}}\{|e_a\rangle|g_b\rangle + |g_a\rangle|e_b\rangle\} \\
|a\rangle &= \frac{1}{\sqrt{2}}\{|e_a\rangle|g_b\rangle - |g_a\rangle|e_b\rangle\}
\end{aligned}
$$

The spontaneous daping accurs via the symmetrical channel $|e\rangle \to |s\rangle \to |g\rangle$ and, respectively, through the antisymmetrical one, $|e\rangle \to |a\rangle \to |g\rangle$ (see Fig.1).

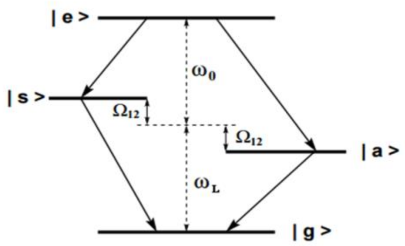

Fig.1 Schematic of the involved two-qubit collective transitions.

Using the master equation (1) one can obtain the following expression for the population of the subradiant state in the absence of the laser field [7]:

$$
\langle R_{aa}(t)\rangle = \frac{e^{-\Gamma_+ t}}{\bar{\Omega}}(\sinh(\bar{\Omega})(2\Gamma(1+\bar{n})R_{ss}(0) + (\Gamma + \gamma\chi_r)\, R_{aa}(0))
$$

$$
+ \bar{\Omega} R_{aa}(0)\cosh(\bar{\Omega}t) + \frac{\gamma R_{ee}(0)e^{-\Gamma_+ t}}{\bar{\Omega}(\Gamma_-^2 - \bar{\Omega}^2)}\,(\bar{\Omega}\cosh(\bar{\Omega}t)\,(\gamma\,(1-\chi_r)^2
$$

$$
-4\Gamma(1+\bar{n}) + \bar{\beta}\,\sinh(\bar{\Omega}t)) + \frac{\gamma R_{ee}(0)e^{-2\gamma t}}{\Gamma_-^2 - \bar{\Omega}^2}\,\,(4\Gamma(1+\bar{n}) - \gamma(1-\chi_r)^2 \qquad (7)
$$

where
$\bar{\beta} = \Gamma_-\{\gamma\chi_r(\chi_r - 1) - \Gamma(3 + \chi_r + 2\bar{n}(1+\chi_r))\} - (1-\chi_r)\bar{\Omega}^2$,
while $\Gamma_{\pm} = \Gamma(1+2\bar{n}) \pm \gamma$, $\bar{\Omega} = \sqrt{\Gamma^2(1+2\bar{n})^2 + \gamma\chi_r(2\Gamma + \gamma\chi_r)}$.
Depending on the initial conditions for $\langle R_{\alpha\alpha}(0)\rangle$, $\alpha \in \{e, s, a, g\}$, and the ratio γ/Γ as well as environmental temperatures one can observe a clear population transfer to the subradiant state. In the absence of phonons and without a cw coherent pumping field, i.e.. $\Gamma = 0$ and $\Omega = 0$ the well-known solution is:

$$
\langle R_{aa}(t)\rangle = \langle R_{aa}(0)\rangle e^{-\gamma(1-\chi_{ab})t} +
+ \frac{1-\chi_{ab}}{1+\chi_{ab}}\{e^{-\gamma(1-\chi_{ab})t} - e^{-2\gamma t}\,\langle R_{ee}(0)\rangle\} \qquad (8)
$$

Finally, in the presence of a laser field, the subradiant state gets populated when the qubits initially are in their ground states. The reason consists in a phonon induced decay rate among the symmetrical and antisymmetrical two-particle states, respectively.

III. CONCLUSIONS

Concluding, we have shortly described the interaction of a laser-pumped two-level qubit pair. The system interacts as well with an environmental electromagnetic field reservoir as well as with the thermostat. Due to the presence of the phonon reservoir, the subradiant state gets populated in the steady state. This occurs in the presence of the laser pumping.

BIBLIOGRAPHY

[1] M. Scheibner, T. Schmidt, L.Worschech, A. Forchel, G.Bacher, T. Passow, and D. Hommel, Superradiance of quantum dots, Nat. Phys. 3, 106 (2007).

[2] T. Brandes, and B. Kramer, Spontaneous emission of phonons by coupled quantum dots, Phys. Rev. Lett. 83,3021 (1999).

[3] V. Ceban, P. Longo, and M. A. Macovei, Fast phonon dynamics of a nanomechanical oscillator due to cooperative effects, Phys. Rev. A 95, 023806 (2017).

[4] R. H. Lehmberg, Radiation from an N-atom system. I.General formalism, Phys. Rev. A 2, 883 (1970).

[5] R. H. Lehmberg, Radiation from an N-atom system. II. Spontaneous emission from a pair of atoms, Phys. Rev.A 2, 889 (1970).

[6] M. Kiffner, M. Macovei, J. Evers, and C. H. Keitel, Vacuum induced processes in multilevel atoms, Prog. Opt.55, 85 (2010).

[7] E. Cecoi, V. Ciornea, A. Isar, M. A. Macovei, Entanglement of a laser-driven pair of two-level qubits via its phonon environment, JOSA B (in print 2018).