Determination of one valued characteristics of power-source and power-load elements

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***Abstract*— The concepts of power-source and power-load elements are considered. The consideration of losses shows the two-valued characteristics of such elements. The interpretation of typical quadratic expressions, as transformations of projective geometry, is used. The projective transformations preserve an invariant in the restricted one-valued area of the characteristic; there is a cross ratio of regime parameters. Direct recalculation formulas with the group properties are obtained. The regime parameters are represented relatively to the characteristic values. The parameter changes are introduced in another way unlike changes in the form of increments. The obtained results are useful for the power supply system monitoring.**

Key words—**Power-source elements, variable parameters, two-valued characteristics, projective transformations.**

I. Introduction

In the electric circuit theory, the concepts of power-source and power-load elements are developed [1- 3]. The consideration of losses of real power-source and power-load elements shows the two-valued volt-ampere characteristic of these elements [4]. Also, the influence of supply line losses onto the power-load element has practical importance [5, 6]. Therefore, regime parameters of the power-load elements are determined reasonably for the one-valued area of their characteristics by using projective transformations and cross ratio for four values of these variable parameters [7]. From here, direct recalculation formulas of the parameters follow. These formulas have the group properties. Also, the regime parameters are represented relatively to the characteristic values; it permits to evaluate the regime effectiveness. Using the obtained results, the regime parameters of the power-source element are determined also in the present paper.

II. Two- Valued Regime of a Regulated Converter. The Concept of a Power-Source and Power-Load Element

Let us consider a simplest power supply system with a voltage source V_0, line resistance R_i, voltage regulator VR and given load resistance R_1 in Fig.1.

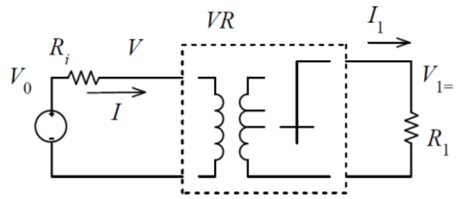

Fig.1. Power supply system with a given load power

The load power

$$P = \frac{(V_{1=})^2}{R_1} = IV \qquad (1)$$

represents a hyperbola in Fig.2. So, we have a power-load element P for different values of voltage V and current I.

In turn, the voltage source characteristic

$$I = \frac{V_0}{R_i} - \frac{V}{R_i}, \qquad (2)$$

intersects the hyperbola into two points M, $\tilde{M}$.

On the contrary, we may say about a power-source element, which gives a constant power into a variable load. In particular, such power supplies are used widely for the fast charging of supercapacitors [8].

Next, we must consider influences of the source and load parameters on the power supply system regime.

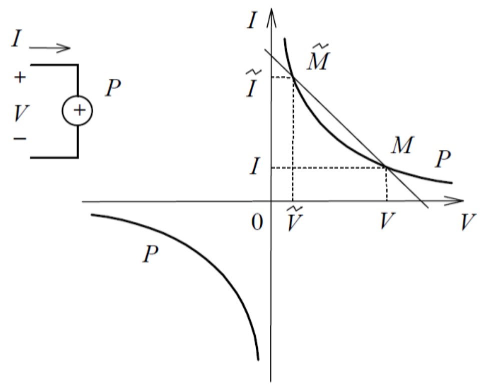

Fig. 2. Power-load element P and its hyperbolic characteristic

III. VOLTAGE OF A POWER SUPPLY WITH LIMITED CAPACITY

Let us consider the simplest power supply system in Fig.3. For different voltage source values (V_0^1, V_0^2), we get the voltage source characteristics as parallel lines. These lines intersect the hyperbola into pairs of points M_1, $\widetilde{M}_1$; M_2, $\widetilde{M}_2$. The arrows show the conformity of these points. Also, the characteristic points M^+, M^- correspond to the tangent lines $\pm V_{0M}$. The closed arrows illustrate these fixed points, as the allowable minimum values

$$V_{0M} = \pm 2\sqrt{R_i P}\ ,\ V_M = \pm\sqrt{R_i P}\ . \qquad (3)$$

We must prove the one-valued restricted area of the power-load element characteristic. To do this, we consider the characteristics of the power-load element in the projective coordinates [9], as it is shown in Fig.4. The parallel lines of the voltage source characteristics are intersected into a point S onto an infinitely remote line ∞. This point S is a pole and the straight line M^+M^- is a polar. Therefore, we get some symmetry or mapping of the "lower" part of our curve onto the "upper" part. So, the obtained one-valued area involves the characteristic points $V_M, \infty, -V_M$.

Now, using the voltages V, V_0 conformity, we may represent the regime parameters relatively to the characteristic values. From (1, 2), we get

$$V_0 = \frac{R_i P}{V} + V\ . \qquad (4)$$

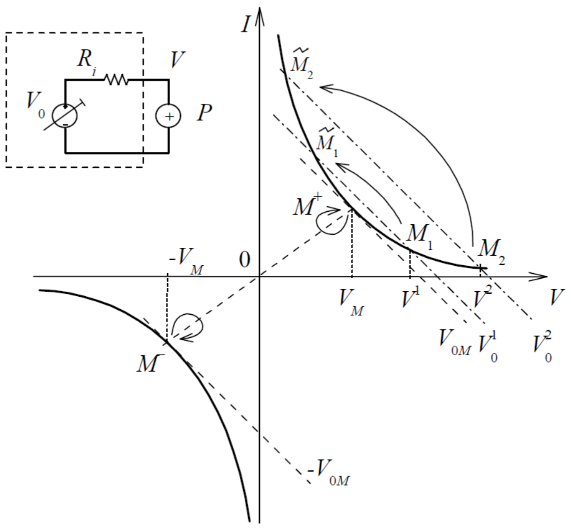

Fig. 3. Characteristics of a power-load element with different values of a voltage source

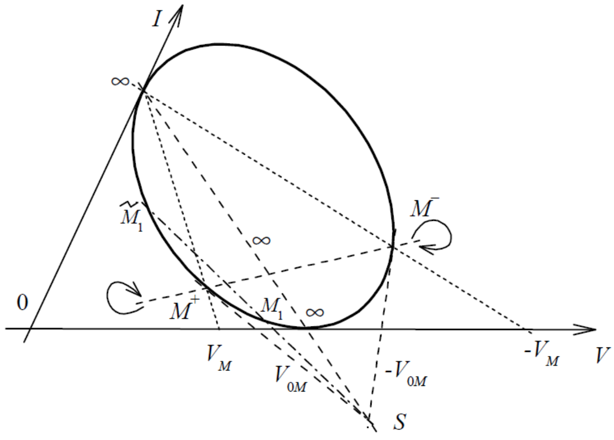

Fig. 4. Characteristics of the power-load element in the projective coordinates

This quadratic dependence determines a hyperbola in Fig.5. So, the one-valued mapping of the hyperbola points onto the axis V takes place. Therefore, we may constitute the cross ratio m_V^1 for the initial point V^1

$$m_V^1 = \left(V_M\ \ V^1\ \ \infty\ \ -V_M\right) = \frac{V^1 - V_M}{V^1 + V_M}\,. \qquad (5)$$

The points $V_M = \dfrac{V_{0M}}{2}$, $-V_M = -\dfrac{V_{0M}}{2}$ are base ones and point $V = \infty$ is a unit point. The conformity of the points V , m_V is shown in Fig.6.

We use the normalized values

$$\overline{V}^2 = 2\frac{V^2}{V_{0M}} > 1, \overline{V}^1 = 2\frac{V^1}{V_{0M}} > 1, \qquad (6)$$

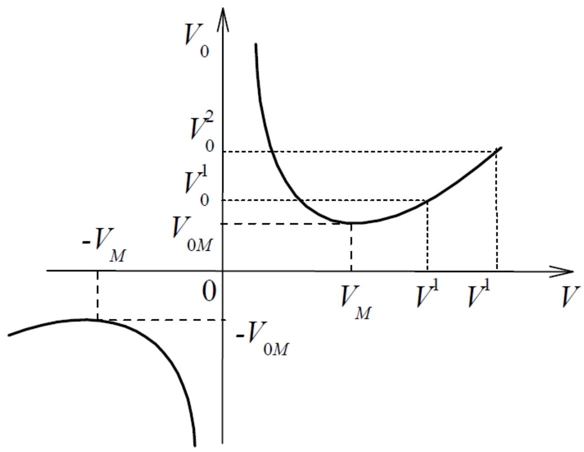

Fig. 5. Dependence $V_0(V)$ for a given load power

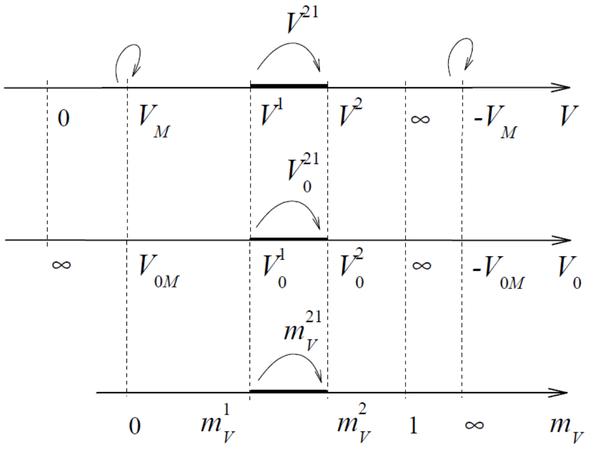

Fig. 6. Conformity of different regime parameters

The regime change $V^1 \to V^2$ are introduced as

$$m_V^{21} = \left(1 \;\; \bar{V}^2 \;\; \bar{V}^1 - 1\right) = \frac{V^{21}+1}{V^{21}-1}. \tag{7}$$

Therefore, the voltage change is determined by

$$V^{21} = \frac{\bar{V}^2 \bar{V}^1 - 1}{\bar{V}^2 - \bar{V}^1}. \tag{8}$$

In turn, the subsequent voltage value

$$\bar{V}^2 = \frac{V^{21}\bar{V}^1 - 1}{V^{21} - \bar{V}^1}. \tag{9}$$

There is a group transformation. If the initial value $\bar{V}^1 = 1$, the subsequent value $\bar{V}^2 = 1$ regardless of the value V^{21}.

Similarly to the above, let us consider the cross ratio m_0^1 for the voltage V_0 using the conformity of the variables for the one-valued area. Then

$$m_0^1 = \left(V_{0M} \;\; V_0^1 \;\; \infty \;\; -V_{0M}\right) = \frac{V_0^1 - V_{0M}}{V_0^1 + V_{0M}}. \tag{10}$$

Using the normalized values

$$\bar{V}_0^2 = \frac{V_0^2}{V_{0M}} > 1, \bar{V}_0^1 = \frac{V_0^1}{V_{0M}} > 1,$$

we get the regime change $V_0^1 \to V_0^2$ similarly to (7)

$$m_0^{21} = \left(1 \;\; \bar{V}_0^2 \;\; \bar{V}_0^1 \;\; -1\right). \tag{11}$$

Also, we introduce the voltage source change

$$V_0^{21} = \frac{\bar{V}_0^2 \bar{V}_0^1 - 1}{\bar{V}_0^2 - \bar{V}_0^1}. \tag{12}$$

The subsequent voltage value

$$\bar{V}_0^2 = \frac{V_0^{21}\bar{V}_0^1 - 1}{V_0^{21} - \bar{V}_0^1}. \tag{13}$$

There is a group transformation also. The base points $V_0, -V_0$ and points $V_M, -V_M$, as the limit points, correspond to the infinitely large distance. Therefore, this limit regime has a clear physical sense.

So, *there is a strong reason to introduce the voltage changes (or system parameters) as* V_0^{21} *and* V^{21}.

Therefore, the important equalities for practice take place

$$m_0^1 = (m_V^1)^2, \; m_0^{21} = (m_V^{21})^2. \tag{14}$$

These equalities are using for measurement and recalculation of variable regime parameters.

IV. Power Source Element with Limited Capacity

Let us consider the simplest power supply system in Fig.7. A voltage source, resistance, power-load element and so on determine the variable load. The volt-ampere characteristics have the view

$$V_0(I) = \frac{P_0}{I}, \; V(I) = \frac{P_0}{I} - IR_i.$$

These characteristics represent corresponding hyperbolas and characteristic $V(I)$ is the two-valued curve.

In turn,

$$V(V_0) = V_0 - \frac{R_i P_0}{V_0}. \tag{15}$$

Though this formula is similar to dependence (4), but the received hyperbole is the two-valued curve in Fig.8.

Now, we must determine one-valued representation of such characteristic. To do that, we may use quadratic values of the voltages (15).

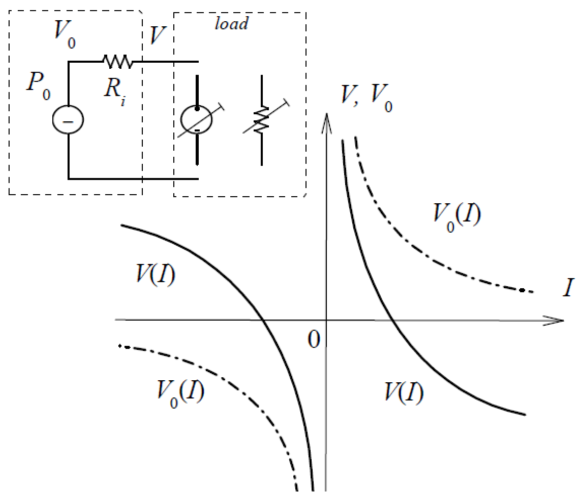

Fig. 7. Characteristics of a power-source element with a variable load

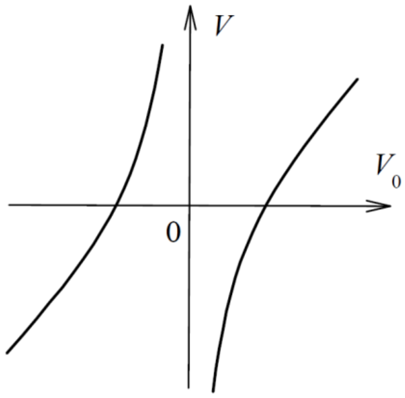

Fig. 8. Voltage characteristic of a power-source element

Then

$$V^2(V_0{}^2) = R_i{}^2 \frac{P_0{}^2}{V_0{}^2} + V_0{}^2 - 2R_i P_0 .$$

This dependence determines a hyperbola in Fig.9. The obtained hyperbola is similar to Fig.6. So, the one-valued mapping of the hyperbola points onto the axis V_0 takes place.

Analogously to (5, 10), we may constitute the cross ratios m_V^1 for the initial points V^1, V_0^1

$$m_V^1 = \left(0\ (V^1)^2\ \infty\ \ -4R_i P_0\right) = \frac{(V^1)^2}{(V^1)^2 + 4R_i P_0},$$

$$m_{V0}^1 = \left(R_i P_0\ (V_0^1)^2\ \infty\ \ -R_i P_0\right) = \frac{(V_0^1)^2 - R_i P_0}{(V_0^1)^2 + R_i P_0}.$$

Using the above results (7, 8, 11, 12), we may introduce the regime and voltage changes. Also, the important equalities for practice take place

$$m_V^1 = (m_{V0}^1)^2,\ m_V^{21} = (m_{V0}^{21})^2 .$$

The conformity of the points V^1, m_V^1 is shown in Fig.10.

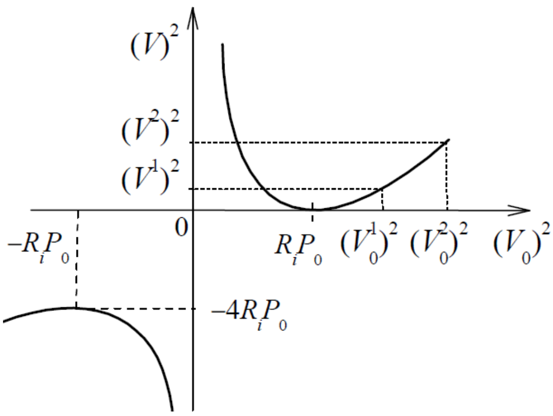

Fig. 9. Quadratic dependence $V^2(V_0{}^2)$ for a given source power

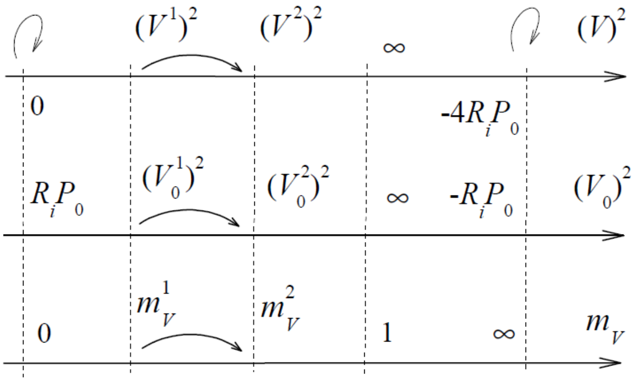

Fig. 10. Conformity of different regime parameters

V. Conclusion

Quadratic characteristics determine invariants as the cross ratio of four regime parameter values. These invariants are executed in a restricted area of regime characteristics. The concrete kind of a power supply system imposes the requirements to definition of already system parameters.

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