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APPLICATION OF THE FINITE DIFFERENCE METHOD TO CALCULATION OF DYNAMIC PROCESSES IN LONG ELECTRICAL LINES

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Abstract. The electrical circuit structure includes components with concentrated and distributed parameters. This creates difficulties in analyzing the dynamic processes, which are described by differential equations, including those with partial derivations. This paper deals with the process of applying equations with partial derivatives (telegraph equations) for the analysis of transient processes using the numerical method of uninterrupted computation. The initial and limit conditions needed to obtain numerical solutions of the transient processes in circuits with the complex variable load have been formulated. The original calculation procedure of the nonstationary regime has been proposed using the finite difference method. The calculation method uses the Riemann invariants. The electrical circuit with lumped parameters includes active resistance, inductance and capacitance. The relative units system was used for the analysis and the procedure of transformation the dimensional parameters among the nondimensional parameters into the relative units system was proposed. The continuous energy storage regime in the ideal inductance in the moving wave regime at the connection of the line end with distributed parameters at the ideal inductance (without losses) with lumped parameters was found.

Keywords: *circuit, concentrated and distributed parameters, initial and limit conditions, mathematical model, telegraph equations.*

Introduction

Electric lines are the basic functional elements of modern power systems with the electrical energy. In these elements phenomena and propagation regimes of the voltage and current waves are manifested [1, 2], which can be conditioned both by internal and external factors. Wave phenomena occur most frequently in high voltage transmission lines which are of extensive lengths [3]. The propagating wave phenomena can also be seen in short electric lines in the form of transient regimes. These are conditioned by the internal (switching, load jumping, short-circuiting) [4] and external factors (atmospheric discharges [5, 6], geomagnetic interference [7]).

The propagation velocity of electromagnetic excitations is high and has the value close to that of the speed of light. This peculiarity creates difficulties for the experimental

observation of the dynamic process in the initial stage of the transitional process, which has the multi-reflective propagation wave mechanism. The reflection effects of the incident waves are conditioned by the existence of nonhomogeneities in the circuits with distributed parameters. The electrodynamic processes, perceived as transient, can seriously damage the power systems by suspending the electricity supplies. The high speed of movement, the nonhomogeneities of the structure of the actual electrical circuits create difficulties for the theoretical analysis of the rapid processes in the electrical circuits. The theoretical calculation both of the permanent and dynamic (transient) regimes in the electrical networks with multiple nonhomogeneities comes across many difficulties. These difficulties are conditioned both by the topological features and the variety of parameter values at the analyzed circuits, as well as the difficulties of using the known calculation methods.

1. Formulation of the Problem within the Study

The nonhomogeneity of the electrical circuit results from the arbitrary intermediate connection of the lumped-parameter load with the electrical network and, at present, at the connection of electricity generation distributed sources with the supply network, inclusive of the renewable energy sources. These peculiarities lead to the need of adapting the equivalent schemes of the analyzed circuits, to the requirements determined by the methods (analytical or numerical) selected to solve the formulated problems. As a reasonable solution, the decomposition of the nonhomogeneous calculation scheme structure into parts with homogeneous properties, the formulation of the boundary conditions and initial conditions at the boundaries of the circuit portions, being considered as homogeneous portions, is used.

In this context, the nonhomogeneous circuit with distributed parameters can be presented in the form of a homogeneous circuit located between the points with large gradient changes of the line parameter values or jump changes of the values of the loads with lumped parameters. The essential changes in current and voltage values in different portions of the circuit can also be conditioned by the energy portions injected in the circuit under the influence of external phenomena, for example, the evolution in time of the electrical and/or magnetic field intensity outside the circuit. Thus, the problem of calculating and analyzing the dynamic regime in non-homogeneous circuits is complex and is a non-trivial task.

In circuits with the distributed parameter, voltage and current values in different circuit sections depend on the values of the leakage capacitive and active currents into the ground. As a result of these peculiarities, for the description of the processes in this type of circuits, we use the equations with partial derivatives (telegraph equations) which are completed with initial and boundary conditions.

The purpose of this paper is to elucidate the possibilities of the finite difference method for the analysis of transient regime evolution peculiarities and wave's propagation in the nonhomogeneous electrical lines with distributed parameters.

2. Basic Equations of the Circuit with Distributed Parameters

The differential equations with partial derivatives are used for elaboration of various mathematical models of physical objects [8], including the energetic domain [9, 10]. Using mathematical models makes it possible to solve multiple applied problems in the field of technology through mathematical simulations [5, 8, 10].

The complex character of the transient processes and the wave processes in the non-homogeneous electric circuits directly influence the selection and applying of the methods of analysis of these dynamic processes. The peculiarities of the calculation procedure are manifested by the fact that the separate examination in time of the transitional and stationary regimes is used. The procedure is used whereby the electrical circuits of the supply system are most often presented by equivalent schemes with the lumped parameters [11, 12].

The development of computers and software allows a new approach to the problem of regime research in circuits with lumped and distributed parameters compared to the known classical methods, which have a long history of use [10, 13, 14].

2.1. The Mathematical Model of the Homogeneous Line

The power source (generator), load (receiver) and the connection circuit (power line) of the source with the load are the main functional components of an electrical power system. The equivalent diagram of an electrical power supply circuit is shown in Figure 1. The source of generation and the load are connected via the electric transmission line with distributed parameters (L , C , R , G).

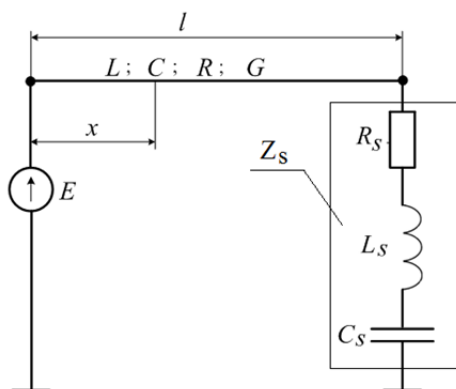


Figure 1. Equivalent scheme of a circuit with distributed parameters L , R , C , G (line length l) and load with lumped parameters R_s , L_s , C_s with voltage source E .

Generally, the processes in the electrical line are described by equations with partial derivatives, which in the electric lines theory are referred to as the "telegraph equations":

$$-\frac{du}{dx} = L \frac{di}{dt} + Ri = 0; \quad -\frac{di}{dt} = C \frac{du}{dt} + Gu = 0; \quad (1)$$

where for linear circuits, the coefficients L , R , C , G are with the constant value. These coefficients are defined as follows: L is the inductance of the loop formed by the line conductor and the conductor for the return current from the load to the source of generation; R is the active longitudinal resistance of the direct conductor; C , G are the capacity and active conductivity of the insulation between the conductor of the circuit and the ground.

The term "telegraph equations" is derived from the "l'equation des telegraphistes", which was suggested by A. Poincaré in 1897.

To obtain the unique solution Eq. (1) has to be filled in with initial and limit conditions. The linear electric circuit, which is shown in Fig.1, at $t = 0$ is connected to the external source E with the arbitrary voltage form:

$$u = U_0(t) \text{ for } x = 0, \quad (2)$$

and the output (end) of the line is connected to an active-reactive load presented by the *RLC* circuit:

$$u_s = R_s i + L_s \frac{di}{dt} + \frac{1}{C_s} \int_0^t i_s(\tau) d\tau, \text{ when } x = l, \quad (3)$$

where U_0 is the voltage at the input terminals of the circuit with distributed parameters. If the power supply E an ideal source (the internal impedance of the source is zero) is met under the condition $E = U_0$; R_s ; L_s , C_s are the lumped parameters of the load; L , R , C , G are the values of the distributed parameters of the line; u_s is the voltage drop on the load; i_s is the load current. The limit regimes for these circuits are the idle mode (IM) and the short-circuit (SC) mode. These boundary marginal conditions correspond to the load impedance values Z_s for the idle regime ($Z_s \rightarrow \infty$) and for the short circuit regime ($Z_s \rightarrow 0$). These factors also indicate the complexity of defining the value of Z_s of the load, even in the case of a homogeneous circuit with distributed parameters. Short circuit regime (SC) and idle regime (IM) can be referred to as the edge limits (or degraded regimes) of the circuit. The SC and IM regimes are not considered as the working regimes of the circuit, but they are useful to be studied as an intermediate step towards the working regimes of the circuits, when the load is modified within the limits of $0 < Z_s < \infty$. Most often, the initial conditions are considered to be null (up to the circuit switching with no electrical charge). Connecting the loads at the intermediate points of line (e.g., $x = x_n$), the connection of the reactive power compensation devices and other equipment to ensure the transmission regime, the currents and voltages as functions of the space variable x can change values by jump at these points of the line.

It is noteworthy that the circuit described by the integral-differential of Eq. (1) can be connected to the line in any place and to ensure structural homogeneity, and the same unique procedure for defining the boundary conditions, the current and voltage through the circuit can be written as $i = i_1 - i_2$ and $u = u_1 - u_2$, where the indices of the currents i_1 , i_2 and the stresses u_1 , u_2 correspond to the values of these functions on the left (index 1) and right (index 2) of the connection point of the *RLC* circuit or of the sources of distributed energy to the line. The load described by Eq. (3) can be represented by the equivalent circuits to the formulation of the limit conditions at the point of connection to the line with distributed parameters (using the concept of decomposition and equation of the circuit portions). In this case, the boundary conditions become more complex than the definition form, but the requirement remains that these *RLC* circuits are linear circuits, because only upon this condition there will be the unique solution of the system of equations describing the circuit under study.

Starting from general theory [15] and referring to Eq. (1), it makes possible to obtain the expression for the integral of energy, which is based on the law of conservation of energy:

$$Li \frac{\partial i}{\partial t} + i \frac{\partial u}{\partial x} + Ri^2 + Cu \frac{\partial u}{\partial t} + u \frac{\partial i}{\partial x} + Gu^2 = 0; \quad \frac{1}{2} \frac{\partial}{\partial t} (Li^2 + Cu^2) + Ri^2 + Gu^2 + \frac{\partial}{\partial x} (iu) = 0. \quad (4)$$

The integration of Eq. (4) into the definition domain $0 \leq x \leq l$ and time interval $0 \leq \tau \leq t$, taking into account the initial zero conditions, enables us to obtain the following equation:

$$\int_0^t \int_0^l (Ri^2 + Gu^2) dx d\tau + \frac{1}{2} \int_0^l (Li^2 + Cu^2) dx = \int_0^t [i(0, \tau)u(0, \tau) - i(l, \tau)u(l, \tau)] d\tau. \quad (5)$$

Equation (5) shows the energy balance of the analyzed circuit. The left-hand side of Eq. (5) is the sum of the active power component in the circuits with distributed parameters (irreversible transformations) and of the reactive power component (reversible energy transformations). The irreversible transformation shows the energy losses in the longitudinal active resistivity $R_l = R l$ and in the active conductivity $G_l = G l^{-1}$ of the insulation of the line portion with the length l which turns into heat. The reversible component is determined by capacity $C_l = C l$ and line inductance $L_l = L l$. The reversible component is the share of the electrical energy, which, consequently, with a double frequency (over a period of the alternating current) is accumulated in the electric field, and then in the magnetic field of the circuit with distributed parameters. The right-hand side of Eq. (5) represents the difference between energy supplied in line and the energy absorbed by the load connected with this line.

All components of Eq. (5) are designated in J (joule) units. It is obvious that for the initial and boundary zero conditions, the integral of Eq. (5) is valid only for the trivial solution. Thus, the theorem of uniqueness results from Fredholm's alternative, which is based on the hypothesis of the existence of the solution for the examined equations [10].

2.2. The Telegraph Equations and Maxwell equations

The telegraph equations adapted for the analysis of the dynamic process in the circuits with distributed parameters were obtained as a result of solving the telephony problem and, therefore, of transmitting electrical signals over long distances. These equations were obtained by examining a closed loop with the right length dx [16].

Equation (1) can be represented as the relationships, which include only one unknown variable, i.e., the current or voltage [17]:

$$\begin{aligned} \frac{\partial^2 i}{\partial x^2} &= LC \frac{\partial^2 i}{\partial t^2} + (LG + RC) \frac{\partial i}{\partial t} + RGi; \\ \frac{\partial^2 u}{\partial x^2} &= LC \frac{\partial^2 u}{\partial t^2} + (LG + RC) \frac{\partial u}{\partial t} + RGu. \end{aligned} \quad (6)$$

The telegraph equations are derived from the Maxwell equations [18] used in electrodynamics:

$$\begin{aligned} \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} &= \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}; \\ \frac{\partial^2 \vec{H}}{\partial x^2} + \frac{\partial^2 \vec{H}}{\partial y^2} + \frac{\partial^2 \vec{H}}{\partial z^2} &= \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}, \end{aligned} \quad (7)$$

where, \vec{E}, \vec{H} are the vectors of the intensity of the electrical field and the magnetic field; μ_0, ϵ_0 are the magnetic and dielectric permeability constants; x, y, z are the spatial coordinates; and t is time.

Comparison of Eq. (6) and Eq. (7) indicates that the telegraph equations are approximate mathematical expressions because they do not take into account the electromagnetic oscillations around the conductor. This observation, which refers to similarity of Eq. (6) and Eq. (7), allows us to apply the known solutions (obtained in the 18-th century by the mathematicians D. Bernoulli, J. D'Alembert and L. Euler) as the structure equation of the following type:

$$a^2 \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial t^2}. \quad (8)$$

Equation (8) describes the oscillation process in the rope. This solution presents the function as the process of propagation of two arbitrary shapes of waves moving in space in the opposite directions with the velocity a :

$$f(x, t) = f_1(x + at) + f_2(x - at). \quad (9)$$

Equation 9 shows the essence of the Method of Characteristics for solving equations with partial derivatives applied to obtain solutions of the first-order equations with partial derivatives. However, it also can be used to solve the hyperbolic equations of a higher order. This method consists in reducing the differential equation with partial derivatives to a family of ordinary differential equations. To use this method it is necessary to determine some curves (hereinafter the term is referred to characteristics) along which the differential derivative of the equation with partial derivatives is transformed into an ordinary differential equation. Thus, the ordinary differential equations can be obtained. The solutions for those equations are known to be the ones in the direction of the straight lines that we call features. The solution found can be transformed into a solution of the initial equation with the partial derivative.

2.3. Report on Characteristics

The nonhomogeneities of the electrical circuit lead to difficulties in the use of calculation methods, including the characteristic methods. The effects of energy dissipation in the circuit (energy losses) also cause difficulties for the realization of the wave process calculations. At present, the efficiency of the electricity transmission was found to be 97%. It is reasonable to use the finite difference method for the dynamic process of calculation in long lines, because the latter are very close to the ideal (lossless) circuits. Based on this hypothesis, Eq. (1) can be simplified (since $R = G = 0$), as follows:

$$-\frac{\partial u}{\partial x} = L \frac{\partial i}{\partial t}; \quad -\frac{\partial i}{\partial x} = C \frac{\partial u}{\partial t}. \quad (10)$$

Equation (10) can be reduced to relations with a single variable by way of multiplying the first equations by the coefficient $\sqrt{\frac{L}{C}}$:

$$\sqrt{\frac{C}{L}} \frac{\partial u}{\partial x} + \sqrt{LC} \frac{\partial i}{\partial t} = 0. \quad (11)$$

By adding the second equation of Eq. (10) to Eq. (11), and by subtracting Eq. (11) from Eq. (10), after transforming these relations we obtain the following expressions:

$$\begin{aligned} \frac{\partial}{\partial x} \left(i + \frac{C}{\sqrt{LC}} u \right) + \frac{1}{\sqrt{LC}} \frac{\partial}{\partial t} \left(i + \frac{C}{\sqrt{LC}} u \right) &= 0; \\ \frac{\partial}{\partial x} \left(i - \frac{C}{\sqrt{LC}} u \right) - \frac{1}{\sqrt{LC}} \frac{\partial}{\partial t} \left(i - \frac{C}{\sqrt{LC}} u \right) &= 0. \end{aligned} \quad (12)$$

The components of Eq. (12) will be as follows: $I^+ = i + \frac{C}{\sqrt{LC}} u$ and $I^- = i - \frac{C}{\sqrt{LC}} u$.

Equation (12) is transformed as follows:

$$\frac{\partial I^+}{\partial x} + \frac{1}{\sqrt{LC}} \frac{\partial I^+}{\partial t} = 0; \quad \frac{\partial I^-}{\partial x} - \frac{1}{\sqrt{LC}} \frac{\partial I^-}{\partial t} = 0. \quad (13)$$

Because $\frac{1}{\sqrt{LC}} = a$, relations of Eq.(13) will be transcribed as following:

$$\frac{\partial I^+}{\partial x} + a \frac{\partial I^+}{\partial t} = 0; \quad \frac{\partial I^-}{\partial x} - a \frac{\partial I^-}{\partial t} = 0. \quad (14)$$

Equation (14) consists of two independent differential equations, which contain only one unknown function $I^+(x,t)$ and $I^-(x,t)$. This system belongs to the hyperbole differential equation class.

The solutions of the differential equations of this system are the following [19]:

$$I^+(x,t) = f^+(x-at); \quad I^-(x,t) = f^-(x+at).$$

If we replace the functions $I^+(x,t)$ and $I^-(x,t)$ by the values u and i , we shall obtain:

$$i + aCu = f^+(x-at); \quad i - aCu = f^-(x+at). \quad (15)$$

Equation (15) allows us to determine the values of the unknown functions i and u from Eq. (10):

$$i = \frac{f^+(x-at) + f^-(x+at)}{2}; \quad u = \frac{f^+(x-at) - f^-(x+at)}{2aC}. \quad (16)$$

The straight lines corresponding to the functions $x - at = const$ and $x + at = const$ in the plane of the variables $x-t$ are referred to as the characteristics of Eq. (14) [19]. For $t = 0$ the x values of the linear equations become equal to the respective constants $x_{t=0}^+$ and $x_{t=0}^-$. Within the range $x^- < x < x^+$, a set of straight lines can be drawn that differ only by the values of the constants of their equations. These constants are identifiers of the straight lines and can be used as the "number" of identification for these straight lines. According to this property the value of the function $I^+(x,t)$ or $I^-(x,t)$ in the point (x, t) depends only on the "number" of the straight line on which that point is located [19]. In this regard, the function $x \mp at$ displays a number of the respective straight lines.

From Eq. (14), it follows that the functions $I^+(x,t)$ and $I^-(x,t)$ in the direction $x \mp at$ of the straight lines have constant values. From the initial conditions we know the

values of the functions i and u in the range of $x^- < x < x^+$ for $t = 0$. When fulfilling these conditions, the solution of Eq. (10) is uniquely determined within the triangle, one side of which coincides with the interval $(x^-; x^+)$. This triangle is analytically determined by the relationships: $t > 0$, $x - at > ax^-$, $x + at < x^+$, and is called the characteristic triangle.

The function $i \pm aCu$ is referred to as the Riemann invariants. The physical sense of the solution presented by the relationship $i + aCu = f^+(x - at)$ consists in that it indicates the distribution of the Riemann invariance, which propagates without deformation in the direction of the straight line $x - at$ with velocity a . Similarly, the expression $i - aCu = f^-(x + at)$ indicates that the second invariant distribution is placed without deformation to the left in the right-hand direction $x + at$ with velocity a . So, by passing consequently all the points in the range of $x^- < x < x^+$, we obtain the values of all the solutions within the characteristic triangle. The Riemann invariants are also referred to as the report on characteristics. Obviously, the value of this constant ratio in the direction of the characteristics is determined only by the number of the characteristics.

The characteristic triangle formed by the Riemann invariants, which is used to obtain the numerical solution of the linear system with partial derivatives for the portion of the electric line with length l is shown in Figure 2.

This portion is divided into N elementary fractions. For the linear circuit with length l with homogeneous structure, three groups of specific points can be pointed out. These are the points x_0, x_N at the ends of the circuit, with the values $x_0 = 0$ and $x_N = l$, and a set of intermediate points in the range of $0 < x_n < l$.

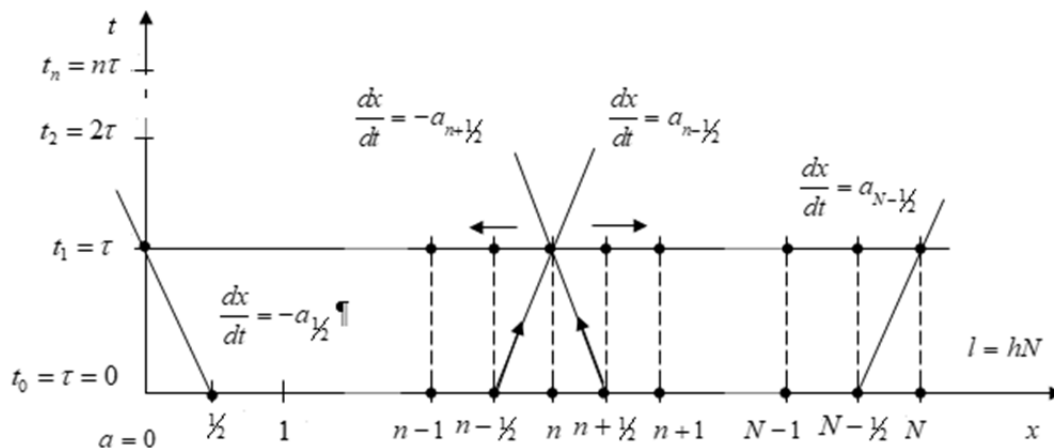


Figure 2. Characteristic triangle for the long line divided into N elementary portions.

2.4. Parameters in the relative units system

The variety of constructive solutions for designing electrical lines, as well as the wide range of load change of these lines, creates difficulties in analyzing their operating regimes.

In order to generalize the results of the calculations of the lines with distributed parameters, it is reasonable to present the values of the parameters of the equivalent scheme of the circuits analyzed in the system of the relative units. In order to achieve this idea it is necessary to select the basic parameters of the analyzed circuit.

As basic parameters it is proposed to use the length of the portion of the circuit with distributed parameters (l), the nominal circuit voltage (U_{nom}), the longitudinal inductance

(L_l), the transverse capacitance (C_l), and characteristic impedance ($Z_{LC} = \sqrt{L/C}$). Absolute values of the inductance and capacitance of the circuit with distributed parameters are calculated using the formulas $L_l = L \cdot l$ and $C_l = C \cdot l$.

The values of the parameters of the examined circuit of Figure 1 in relative units are calculated using the following relations:

$$\begin{aligned} u_0(t) &= \frac{u(t)}{U_{nom}}; \quad i_0(t) = \frac{i(t)Z_{LC}}{U_{nom}}; \quad t_0 = \frac{t \cdot a}{l}; \quad x_0 = \frac{x}{l}; \\ L_0 &= \frac{L \cdot l}{L_l} = 1; \quad C_0 = \frac{C \cdot l}{C_l} = 1; \quad a_0 = 1/\sqrt{L_0 C_0} = 1; \\ R_0 &= \frac{R \cdot l}{Z_{LC}}; \quad G_0 = G \cdot l \cdot Z_{LC}; \quad R_{0S} = \frac{R_S}{Z_{LC}}; \quad L_{0S} = \frac{L_S}{L \cdot l}; \quad C_{0S} = \frac{C_S}{C \cdot l}. \end{aligned} \quad (17)$$

When we use the system of relative units, the equations with partial derivatives, Eq. (1), the initial and boundary conditions can be written as follows:

$$\begin{aligned} \frac{\partial i}{\partial t} + \frac{\partial u}{\partial x} &= 0, \quad \frac{\partial u}{\partial t} + \frac{\partial i}{\partial x} = 0, \quad \text{for } x \in (0, 1), t > 0; \\ u(x, t) &= i(x, t) = 0, \quad \text{for } t = 0, x \in [0, 1]; \\ u(0, t) &= 1, \quad \text{for } x = 0, t \geq 0; \\ u_s(1, t) &= R_s i(1, t) + L_s \frac{di(1, t)}{dt} + \frac{1}{C_s} \int_0^t i(1, \tau) d\tau, \quad \text{for } x = 1, t \geq 0. \end{aligned} \quad (18)$$

in which $L = C = 1$, $R = G = 0$, $l = 1$, $E(t) = U_0(t) = f(t) = 1$ and Eq. (1), initial conditions of Eq. (2) and limit Eq. (3) can be represented by simplistic equations.

As the basic parameter, it is possible to select the wavelength $\lambda = \frac{a}{f}$ of the signals transmitted via the circuit with distributed parameters, with the wave propagation velocity; f is the frequency of the signal transmitted over the circuit.

3. The numerical solution

To obtain a determined solution, it is necessary to formulate additional conditions. These conditions are called initial, which determine in the portion of $x_0 < x < x_N$ for $t \geq 0$ the distribution of current and voltage in the line. If we have a nonhomogeneous circuit, which includes homogeneous portions, for example, which are defined in the intervals $x_{m-1} < x < x_m$, where m is the number of the homogeneous portion, x_{m-1} is the beginning and x_m is the end of the homogeneous portion.

To obtain the numerical solution it is necessary to introduce a uniform network of computation on the continuous variation portion of the argument $x \in [0, l]$ with the value of the meshes h_n , $n = 1, 2, 3, \dots, N$. This network includes whole nodes and half-nodes. The unknown functions of current and voltage are determined in the half-nodes of the numerical calculus of network, which is determined from the relationship $x_{n-1/2} = (x_{n-1} + x_n)/2$. The computing network obtained by dividing the length or the homogeneous portion of the nonhomogeneous line is formed so that the

$h_n / a_{n-1/2} = \tau = const$ condition is met within the partitioned portion. The time discretization step τ is noted in organizing the numerical computation process for obtaining the solution of the differential equation. Concurrently it is necessary to meet the condition that the boundaries of the splitting segments of the homogeneous portions of the line with distributed parameters coincide with the points which represent the whole nodes of the calculation network $x = x_n$.

The calculation is performed in successive steps. Knowing the values of i and u functions over the previous time interval (e.g., from the initial conditions), the values of the functions i and u are determined for the current time interval in the numerical computation process.

Approximation of the first derivatives is carried out using the centered values of the unknown variables.

The transformation of Eq. (1) is carried out by substituting the derivatives of the unknown variables by analogy with the presented in the form of finite differences, thus obtaining the following relations:

$$\begin{aligned} -\frac{u^n - u^{n-1}}{h_n} &= L_{n-1/2} \frac{i^{n-1/2} - i_{n-1/2}}{\tau} + R_{n-1/2} \frac{i^{n-1/2} + i_{n-1/2}}{2}; \\ -\frac{i^n - i^{n-1}}{h_n} &= C_{n-1/2} \frac{u^{n-1/2} - u_{n-1/2}}{\tau} + G_{n-1/2} \frac{u^{n-1/2} + u_{n-1/2}}{2}. \end{aligned} \quad (19)$$

or in an obvious form:

$$\begin{aligned} i^{n-1/2} &= \left(L_{n-1/2} + \frac{\tau}{2} R_{n-1/2} \right)^{-1} \left(\frac{\tau}{h_n} (u^{n-1} - u^n) + (L_{n-1/2} - \frac{\tau}{2} R_{n-1/2}) i_{n-1/2} \right); \\ u^{n-1/2} &= \left(C_{n-1/2} + \frac{\tau}{2} G_{n-1/2} \right)^{-1} \left(\frac{\tau}{h_n} (i^{n-1} - i^n) + (C_{n-1/2} - \frac{\tau}{2} G_{n-1/2}) u_{n-1/2} \right). \end{aligned} \quad (20)$$

The top indices correspond to the "top" time range. Additional values i^n, u^n given in the points x_n are calculated from the linear relationships $dx/dt = \pm a$, in which the constants $I^\pm(x, t) = i \pm aCu$ that correspond to Eq. (20), maintain their constant values: $I^\pm(x, t) = const$.

When accepting the hypothesis that unknown functions have constant values in the range of (x_{n-1}, x_n) , an algebraic system of equations with two unknowns can be obtained i^n, u^n :

$$\begin{aligned} i^n + a_{n-1/2} C_{n-1/2} u^n &= i_{n-1/2} + a_{n-1/2} C_{n-1/2} u_{n-1/2}; \\ i^n - a_{n+1/2} C_{n+1/2} u^n &= i_{n+1/2} - a_{n+1/2} C_{n+1/2} u_{n+1/2}. \end{aligned} \quad (21)$$

From Eq. (21) we can obtain relations for determination of intermediate values of unknown parameters:

To calculate the values of i and u at points $x_0 = 0$ and $x_N = l$ the boundary conditions and the corresponding relations on the characteristics are used:

$$\begin{aligned}
\alpha_0 i_0 + \beta_0 u_0 &= f_0; \\
i_0 - a_{1/2} C_{1/2} u_0 &= i_{1/2} - a_{1/2} C_{1/2} u_{1/2}; \\
\alpha_l i_N + \beta_l u_N &= f_l; \\
i_N + a_{N-1/2} C_{N-1/2} u_N &= i_{N-1/2} + a_{N-1/2} C_{N-1/2} u_{N-1/2},
\end{aligned} \tag{23}$$

where α, β, f are the known functions of the time variable.

For example, the voltage $u(0, t)$, which is described by the function $f_0(t)$, so $u(0, t) = f_0(t)$, has been applied at the beginning of the line (at the coordinate point $x = 0$). At the end of the line the initial values are determined, which are considered to be null for voltage or current: $u(l, t) = 0$ (SC), $i(l, t) = 0$ (IM). These parameters can be replaced at the power consumed on the load as a function of time: $i(l, t) \cdot u(l, t) = P_l(t)$. In this case, the boundary condition is non-linear and the solution to the problem can be undetermined, or not generally determined.

For a conducting medium with values jumps of the distribution parameters L, C, R, G at the connection boundaries of the portions $x = x_n$ the continuity conditions [20] are defined:

$$[i] = [u] = 0 \quad \text{for} \quad x = x_n, t > 0 \tag{24}$$

Using i and u , the conditions were noted that from the left and right of the border point the current or the tension do not change by jumps of the values. This coincides with the principle of continuity of the current in the conductor $u^+ = u^-$ and $i^+ = i^-$, so also, u^+, i^+ where the values of voltage are on the right and u^-, i^- where the voltage and current values are on the left of the border point.

Thus, the initial conditions are defined for two unknown functions i and u , and the boundary condition at the conductor ends is the only one. From the properties of the hyperbolic systems of equations, it is also necessary that α and β satisfy the following relations:

$$\begin{vmatrix} \alpha_0 & \beta_0 \\ 1 & a_0 C_0 \end{vmatrix} \neq 0; \quad \begin{vmatrix} \alpha_l & \beta_l \\ 1 & a_l C_l \end{vmatrix} \neq 0. \tag{25}$$

where a_0, a_l are the propagation velocities of electromagnetic waves at the beginning and end of the line. If the condition of Eq. (25) is satisfied, Eq. (22) has a single solution.

The necessary and sufficient stability condition of the numerical calculation method is obtained by the method of a priori estimation of the energy balance given by the relation [10]:

$$\tau \leq h_n / a_{n-1/2}. \tag{26}$$

If we use point $(x_{n-1/2}; t_0)$ as a reference, the calculated numerical scheme corresponds to the first degree of approximation of the time and linear dimension variables. The analysis of the discrete approximation of the differential Eq. (20) indicates that when using the maximum value of the time $\tau = h_n / a_{n-1/2}$ discretization step, leakage or fictitious sources of energy are not observed. This also refers to the phenomena of dissipation and dispersion of the numerical calculation scheme (in some cases up to zero). This observation is confirmed both by the numerical solutions of the classical problems, for which the

analytical solutions (standard) are known, and in the strict compliance with the energy balance in the line of Eq. (5) to the difference level.

If $\tau > h_n / a_{n-1/2}$ the obvious Eq. (20) – Eq. (23) predictor-correction pattern is unstable, and if $\tau < h_n / a_{n-1/2}$ the difference in energy dissipation can be very essential, and the simulated waveform can be heavily distorted.

The essence of the necessity to use the allowed limit value of the time discretization step τ , arises from the fact, that the slope of the signal, which propagates at speed $a_{n-1/2}$, has to travel the distance equal to the step h_n of the numerical computing network.

In the proposed method, which allows simultaneous determination of instantaneous values of current and voltage at an arbitrary point of the line, the calculation is made using formulas with a typical structure that do not depend on coordinates. The conditions of Eq. (24) are automatically imposed for the contact border sections $x = x_n$, and are not particularly highlights, as is needed for other methods of solving telegraph equations for lines containing portions with different values of distributed parameters. The relationships in the finite differences allow passing to the limit, i.e., if $L \rightarrow 0$ (or $C \rightarrow \infty$) we have $u \rightarrow 0$, and $L \rightarrow \infty$ (or $C \rightarrow 0$) we obtain $i \rightarrow 0$. This way it is easy to consider the existence in the nonhomogeneities the line with the concentrated parameters of bipolar and multipolar types of switches, reactors, filters, phase difference transformers, etc.

The finite difference scheme allows for the calculation of continuous (general) solutions for which the wave fronts are automatically highlighted.

The proposed algorithm is easy to program and allows for extensive parametric investigations of transient and stationary processes, the knowledge of which is necessary for a correct determination of the operating regimes and realization of the technical systems with distributed and localized nonhomogeneities without too much idealization of objects with distributed parameters.

Obtaining results similar to those exhibited by experimental methods is a very complex issue, and it is often impossible to find a satisfactory solution to the problem under consideration.

Based on the invariant character of the wave equations (for the case $R = 0$, $G = 0$) to the substitution procedure $t \rightarrow -t$ and $x \rightarrow -x$ it is possible to inverse the time and restore the initial signal. Reversing the time variable is also permissible for a line with loss and deformation of the signal, namely, in the line with the parameters $R \neq 0$ and $G \neq 0$.

3.1. The Procedure for Calculating Voltage and Current in the Nodes and Half-Nodes of the Line

The solution is obtained in two phases. At the first stage the unknown values of current i and voltage u are determined at the points marked as whole nodes $(x_n, t_0 + \tau)$, taking into account the connection conditions in the section $x_n = const$. For all points denoted as nodes $0 < x_n < l$ in the range the conditions $i_n^- = i_n^+$ and $u_n^- = u_n^+$.

The values of the currents i^n and voltages u^n in the points $x_n = n \cdot h$ are determined from the system of algebraic equations:

$$\begin{aligned} (i^- + aCu^-)^n &= (i^- + aCu^-)_{n-1/2}; & (i^+ - aCu^+)^n &= (i^+ - aCu^+)_{n+1/2} \\ i^- - i^+ &= 0; & u^- - u^+ &= 0, \end{aligned} \quad (27)$$

where h is the step of grid; $n = 0, 1, 2, 3, \dots$ is the order number of the grid point where the unknown values are determined for time $t_i = t_{i-1} + \tau$; τ is the discretization of time in the numerical calculation scheme.

In Eq. (27) index n does not mean raising to a power. This only emphasizes that the current and voltage values are determined for the node to the calculation grid denoted by n . Values of such unknown variables as i^n and u^n are additional intermediate sizes.

The unknown variables in the algebraic equation system of Eq. (27) are designated via the transduced vector of the currents and the voltages $Z = \left((i^-)^n, (i^+)^n, (u^-)^n, (u^+)^n \right)^T$ at the point with the coordinates $(x_n, t + \tau)$ can be determined from the matrix equation:

$$BZ = Y \quad (28)$$

Where

$$B = \begin{vmatrix} 1 & 0 & +aC & 0 \\ 0 & 1 & 0 & -aC \\ 1 & -1 & 0 & 0 \\ 0 & 0 & +1 & -1 \end{vmatrix}; Y = (Y_1, Y_2, Y_3, Y_4)^T; Y_1 = (i^- + aCu^-)_{n-1/2}; Y_2 = (i^+ - aCu^+)_{n+1/2}; Y_3 = Y_4 = 0.$$

Element values Y_i in system equations of Eq. (28) are known because they were calculated for the time $t = t_{i-1}$. To get the solution of the matrix equation we multiply the left- and right-hand parts to the inverted matrix B^{-1} :

$$Z = B^{-1}Y \quad (29)$$

After the values of the Z vector have been obtained, in all nodes " n " of the numerical calculation grid, using Eq.(29), the values of current and voltage in the half-nodes of the numerical grids $(x_{n-1/2}, t + \tau)$ can be calculated by means of Eq. (20).

3.2. Determination of Voltage and Current at the Marginal Points of the Line

3.2.1. The input in line

For a more general overview of the use of the finite difference method, we shall consider that the line ends are connected to circuits with RLC parameters. This hypothesis can describe the power supply source, since any power source is characterized by the internal impedance which may generally include all passive components of the RLC type circuit.

The RLC passive components have the serial connection and the balance of the voltages from the connection points line with distributed parameters and are described by the integral-differential equation $u = Ri + L \frac{di}{dt} + \frac{1}{C} \int_0^t idt$.

At the line input the conditions $i^+ = i$ and $u^+ = u$ are fulfilled. The voltage equilibrium at the line input is represented by the following relations:

$$R_E i + L_E \frac{di}{dt} + \frac{1}{C_E} \int_0^t i dt - u_E = 0$$

or in finite differences

$$R_E i^n + L_E \frac{i^n - i_n^+}{\tau} + \frac{\tau}{C_E} (i^n + i_n^+ + i_{n-1}^+ + \dots) - u_E = 0.$$

By transforming the last expressions we get the relationship:

$$\left(R_E + \frac{L_E}{\tau} + \frac{\tau}{C_E} \right) i^n - u_E = \frac{L_E}{\tau} i_n^+ - \frac{\tau}{C_E} \sum_{k=0}^n i_{n-k}^+,$$

where i^n is the current value for the time step is $t^n = t_{n+1} - t_n + \tau$; i_n^+ is the current value at the previous step of time t_n ; i_{n-k}^+ is the instantaneous value of the current at the previous time; τ is discretization intervals of time t .

At the entry of the line the following system of equations is true:

$$\begin{aligned} (i^+ - aCu^+)^0 &= (i^+ - aCu^+)_{1/2}; \\ \left(R_E + \frac{L_E}{\tau} + \frac{\tau}{C_E} \right) i^+ - u^+ &= \frac{L_E}{\tau} i_n^+ - \frac{\tau}{C_E} \sum_{k=0}^n i_{n-k}^+. \end{aligned} \quad (30)$$

For $k = n$ we obtain the known value of the current i from the initial conditions for $t = 0$, applying the system of equations in the form of finite differences matrix: $BZ = Y$, where

$$B = \begin{bmatrix} 1 & -aC \\ R_E + \frac{L_E}{\tau} + \frac{\tau}{C_E} & -1 \end{bmatrix};$$

$$Z = (i^+, u^+)^T = (i, u)^T; Y = (Y_1, Y_2)^T; Y_1 = (i - aCu)_{1/2}; Y_2 = \frac{L_E}{\tau} i_n - \frac{\tau}{C_E} \sum_{k=0}^n i_{n-k}.$$

Matrix B for marginal regimes is of idle and short circuit regimes. In the idle regime, the coefficient next to the unknown variable $i^+ = i$ in the second equation of (30) tends to $\rightarrow \infty$. If we divide the components of the second equation into the $\left(R_E + \frac{L_E}{\tau} + \frac{\tau}{C_E} \right) = \infty$ coefficient, we get a modified relationship of $1 \cdot i - 0 \cdot u = 0$, because

$$\left(\frac{1}{\left(R_E + \frac{L_E}{\tau} + \frac{\tau}{C_E} \right)} \right) = 0; \frac{\frac{L_E}{\tau} i_n - \frac{\tau}{C_E} \sum_{k=0}^n i_{n-k}}{R_E + \frac{L_E}{\tau} + \frac{\tau}{C_E}} \rightarrow 0. \text{ For these conditions, we obtain the}$$

$$\text{matrices } B = \begin{bmatrix} 1 & -aC \\ 1 & 0 \end{bmatrix} \text{ and } Y_2 = 0.$$

In the short-circuit regime, the voltage $u = 0$ and $i \neq 0$ in the point $x = 0$, the equation has the structure: $0 \cdot i - 1 \cdot u = 0$, because only for $R_E + \frac{L_E}{\tau} + \frac{\tau}{C_E} = 0$, the condition $u = 0$ for the short circuit will be fulfilled. The matrices, which present the coefficients of the independent variables are $B = \begin{vmatrix} 1 & -aC \\ 0 & 1 \end{vmatrix}$ and $Y_2 = 0$.

3.2.2. End of the line

For the end of the line $x = l$, to which the RLC circuit is connected, the equations are valid:

$$(i^- + aCu^-)^N = (i^- + aCu^-)_{N-1/2}; \quad R_S i + L_S \frac{di}{dt} + \frac{1}{C_S} \int_0^l i dt - u_S = 0, \quad (31)$$

where $i = i = i^N$ și $u = u$.

The second equation in finite differences system of Eq. (31) is as follows:

$$R_S i + L_S \frac{i^n - i_n^-}{\tau} + \frac{\tau}{C_S} (i^n + i_n^- + i_{n-1}^- + \dots) - u = 0. \quad (32)$$

After some transformations of equation Eq. (32) we get the relation:

$$\left(R_S + \frac{L_S}{\tau} + \frac{\tau}{C_S} \right) i^n - u = \frac{L_S}{\tau} i_n^- - \frac{\tau}{C_S} (i_n^- + i_{n-1}^- + i_{n-2}^- + \dots).$$

The elements of the matrix equation for point $x = l$ are the following:

$$B = \begin{vmatrix} 1 & aC \\ R_S + \frac{L_S}{\tau} + \frac{\tau}{C_S} & -1 \end{vmatrix}; \quad Y_1 = (i + aCu)_{N-1/2}; \quad Y_2 = \frac{L_S}{\tau} i_n^- - \frac{\tau}{C_S} (i_n^- + i_{n-1}^- + \dots).$$

At idle regime at the end of the line $i = 0$; $u \neq 0$ and $R_S + \frac{L_S}{\tau} + \frac{\tau}{C_S} = \infty$, or after division we get the equation:

$$1 \cdot i - \frac{1}{R_S + \frac{L_S}{\tau} + \frac{\tau}{C_S}} u = \frac{\frac{L_S}{\tau} i_n^- - \frac{\tau}{C_S} (i_n^- + i_{n-1}^- + \dots)}{R_S + \frac{L_S}{\tau} + \frac{\tau}{C_S}}$$

The system of equations of Eq. (31) turns into the following relationship:

$$(i + aCu)^N = (i + aCu)_{N-1/2}; \quad 1 \cdot i + 0 \cdot u = 0,$$

because $B = \begin{vmatrix} 1 & aC \\ 1 & 0 \end{vmatrix}$; $Y_1 = (i + aCu)_{N-1/2}$; $Y_2 = 0$.

In case $R_s + \frac{L_s}{\tau} + \frac{\tau}{C_s} = 0$ of short circuit ($i \neq 0$), from which it follows that we also have $\frac{L_s}{\tau} = 0$ and $\frac{\tau}{C_s} = 0$ for any step τ of the discretization for the time t . The following system of equations is obtained:

$$\begin{aligned}(i + aCu)^N &= (i + aCu)_{N-1/2}; \\ 0 \cdot i - 1 \cdot u &= 0,\end{aligned}$$

where from $B = \begin{vmatrix} 1 & aC \\ 0 & -1 \end{vmatrix}$; $Y_1 = (i + aCu)_{N-1/2}$; $Y_2 = 0$.

3.3. Connecting the long line to the DC voltage source

Analysis of the regimes with the multiple reflection wave becomes difficult for the line with the energy loss and signal distortion. In any electric circuit with distributed parameters, the wave distortion phenomenon is observed when the condition of $CR \neq LG$ is not met. To obtain the solution we shall use the finite difference scheme constructed for the whole node ($n, n-1$) and half-nodes ($n-1/2; n+1/2$) of the grid [17]. In the approximate equations in finite differences of Eq. (19) the α_d, β_d correction coefficients were introduced [21], which decreased the impact of the effects of dissipation and dispersion of the waves in their propagation process in the circuit with the distributed parameters:

$$\begin{aligned}(L + \tau\alpha_d)_{n-1/2} \frac{i^{n-1/2} - i_{n-1/2}}{\tau} + \frac{u^n - u^{n-1}}{h} + (Ri)_{n-1/2} &= 0; \\ (C - \tau\beta_d)_{n-1/2} \frac{u^{n-1/2} - u_{n-1/2}}{\tau} + \frac{i^n - i^{n-1}}{h} + (Gu)_{n-1/2} &= 0,\end{aligned} \quad (33)$$

in which $i^n = \frac{(Z_B i)_{n-1/2} + (Z_B i)_{n+1/2} + u_{n-1/2} - u_{n+1/2}}{Z_{B,n-1/2} + Z_{B,n+1/2}}$; $u^n = \frac{(u/Z_B)_{n-1/2} + (u/Z_B)_{n+1/2} + i_{n-1/2} - i_{n+1/2}}{(1/Z_{B,n-1/2}) + (1/Z_{B,n+1/2})}$;

$$\alpha_d = \left(\frac{R}{2} + \frac{GL}{C} \right)_{n-1/2}; \quad \beta_d = \left(\frac{G}{2} + \frac{RC}{L} \right)_{n-1/2}; \quad \tau = (h/a)_{n-1/2} = \text{const.}$$

In the transcription shown, the numerical calculation scheme of Eq. (33) is characterized by the null of the dissipation of the numerical solution and a minimum value of the dispersion of this solution. The energy balance at the finite difference is strictly observed regardless of the value of the meshing step of the calculation grid.

As a test problem we will examine the fall of the voltage wave on the ideal capacity with $C_s = 80 \mu F$ [22].

The cable line parameters are as follows: $C = 0.25 \mu F / km$; $L = 31.2 mH / km$; $R = 1.88 Ohm / km$; , $G = 0$; $l = 320 km$; $a = 16436 km / s$. The wavelength is $\lambda = a / f = 228.57 km$. These parameter values are selected to allow us the numerical solution with the analytical solution from source [22] to be compared. In the relative units system, the parameters have the following values: $C = L = l = C_s = a = 1, R = 1.703, G = 0$, because at the application of a voltage to the line input, the energy losses in the insulation can be neglected.

3.3.1. Capacitive load

The initial conditions are null and the boundary conditions at the entrance and end of the line are as follows:

$$u = 1 \text{ for } x = 0; \quad u = \frac{1}{C_S} \int_0^t i(\tau) d\tau \text{ for } x = 1 \quad (34)$$

The approximation in the finite difference of the boundary conditions is performed. The currents and voltages at the point with the coordinates $x = l$ are presented by the vector $Z = (i, u)^T$. To determine the parameter Z it is necessary to solve the system from two equations with two unknowns, which in the matrix form can be presented as follows:

$$BZ = Y, \quad (35)$$

$$\text{where } B = \begin{vmatrix} 1 & aC \\ \tau/C_S & -1 \end{vmatrix}; \quad Y_1 = i_{n-1/2} + aCu_{n-1/2}; \quad Y_2 = -\frac{\tau}{C_S} \left(i_n + i_n^v + \dots \right).$$

From these formulas we can see that in this case we have the short-circuit mode. From these formulas we can observe that for $C_S = \infty$ we have $u = 0$, and in this case we have the short-circuit mode as well.

The variation of voltage over time (u_c curve) and current (i_c) at the end of line with the capacitive load are shown in Figure 3. A large number of calculation experiments have been performed which show that in the proposed numerical scheme the effects of wave dispersion and dissipation are minimal even for the grid with high values of the eyes, which has about 10-20 knots. The numerical solutions obtained are practically correct, because there is an exact energy balance at the scheme level in finite differences. Also, a good internal convergence of the calculation method is observed, in the case of simulating the propagation process of the rectangular wave.

The curves analysis of Figure 3 on the capacitor connected at the end of the cable line indicates that changes through jump (curve i_c) occur at a time that corresponds to that of arrival of the voltage wave u_c at the condenser connection point of the line in the cable. The amplitude of the wave decreases with each passage in the cable due to energy dissipation.

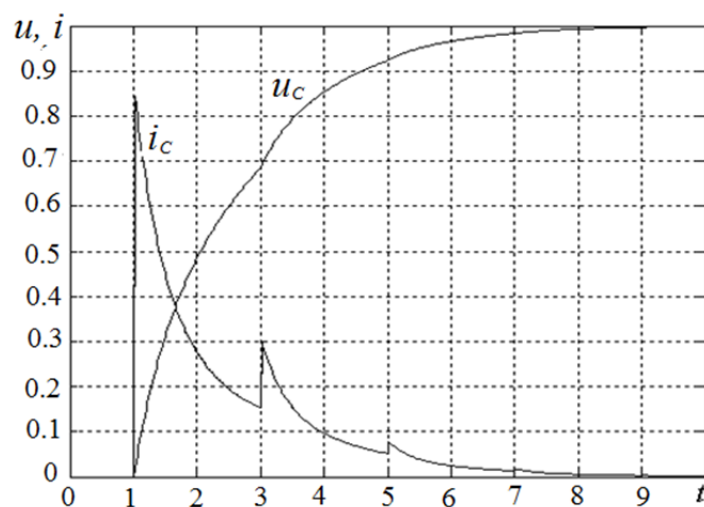


Figure 3. The voltage (curve 1) and the current (curve 2) at the end of the line at $C = L = a = l = 1$; $R = 1.703$; $G = 0$; $C_S = 1$; $R_S = L_S = 0$.

3.3.2. Active load

If the line in the cable has an active load $R_S = 1.703$, for this load the boundary conditions will be determined by the relation of $u = R_S i$ for $x = l = 1$. Matrix equation $BZ = Y$ will have the following coefficients:

$$B = \begin{vmatrix} 1 & aC \\ R_S & -1 \end{vmatrix}; \quad Y_1 = i_{n-1/2} + aCu_{n-1/2}; \quad Y_2 = 0.$$

For parameter values of the tested cable and load with active value $R_S = 1.703$ r.u. a rapid attenuation of the magnitude of the voltage and current waves is observed with the achievement of the permanent regime of Figure 4.

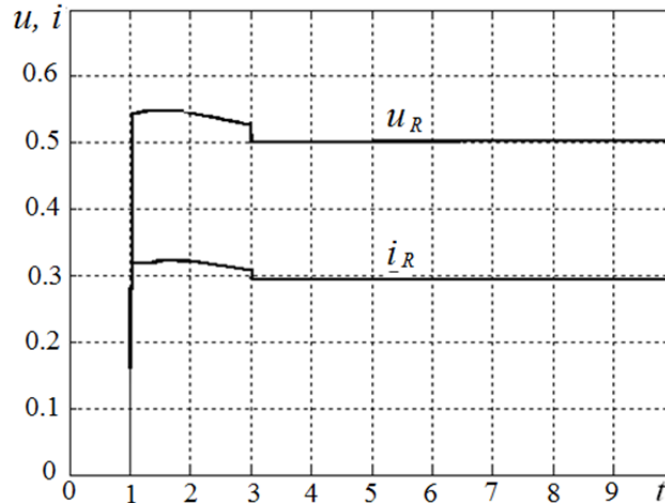


Figure 4. The voltage (curve u_R) and current (curve i_R) at the end of line for parameters of the circuit: $C = L = a = l = 1$; $R = 1.703$; $G = 0$; $C_S = \infty$; $R_S = 1.703$; $L_S = 0$.

3.3.3. Inductive load

Let us examine the case when the cable line has an inductance without loss of $L_S = 1$ value. The L_S inductance voltage drop is determined from the relationship $u = L_S \frac{di}{dt}$ for $x = 1$ using the matrix equation of Eq. (35). The coefficients in Eq. (35) are determined by the following relationships:

$$B = \begin{vmatrix} 1 & aC \\ L_S / \tau & -1 \end{vmatrix}; \quad Y_1 = i_{n-1/2} + aCu_{n-1/2}; \quad Y_2 = \frac{L_S}{\tau} i_n.$$

The voltage and current evolution curves on the inductive load of the cable line obtained by numerical calculation are shown in Figure 5a.

Parametric analysis of the wave processes in the line without losses with inductive load indicates the increase in the current of the circuit, while the inductance voltage has an oscillatory character (Figure 5b).

In course of time, the amplitude of the voltage decreases and the circuit switches to short circuit.

The observed process may be of practical interest, for example, if it is necessary to solve the problem of storing energy in the inductance with superconductivity from a source with a limited voltage value.

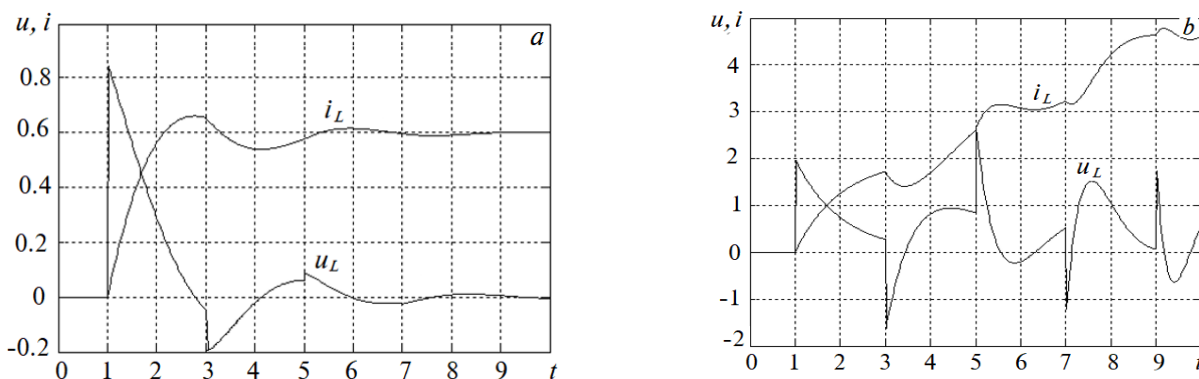


Figure 5. Curves of current i_L and voltage u_L on inductance without losses L_S connected to the line in cable: $C = L = a = l = 1$; $R = 1.703$; $G = 0$; $L_S = 1$; $C_S = \infty$; $R_S = 0$ (a) and $R = 0$; $G = 0$; $L_S = 1$; $C_S = \infty$; $R_S = 0$ (b).

3.3.4. RLC load

The limit conditions at the end of the line ($x = 1$) are represented by the integral-differential equation: $u_s = R_S i + L_S \frac{di}{dt} + \frac{1}{C_S} \int_0^t i(\tau) d\tau$.

At the finite difference approximation of Eq. (35) we obtain the following relations for its coefficients:

$$B = \begin{vmatrix} 1 & aC \\ b_{12} & -1 \end{vmatrix}; \quad Y_1 = i_{n-1/2} + aC u_{n-1/2}; \quad Y_2 = \frac{L_S}{\tau} i_{n-1/2} + \frac{\tau}{C_S} \left(i_n + i_n^v \right); \quad b_{12} = R_S + \frac{L_S}{\tau} + \frac{\tau}{C_S}.$$

It can be observed that when the condition of $R_S = L_S = 0$ is met and $C_S = \infty$ the short-circuit mode ($u_s = 0$) is obtained, then for the conditions of $L_S = 0$ and $C_S = R_S = \infty$ we have the idle regime, because the current $i = i_s = 0$. Figure 6 shows the voltage and current curves in the line load. In Figure 6a, the cable line insulation losses are considered equal to zero, so $G = 0$. The line energy losses are determined only by its longitudinal resistance R . In Figure 6b, the voltage and current curves in the Z_S load are also influenced by the active conductivity G of the line, that is to say, the losses in cable line insulation. These results indicate the influence of line losses on the characteristics of the transient process in the examined circuit, as the increase of the leakage currents in the RLC load decreases. This is an obvious finding, but the originality of the solution obtained is determined by the fact that it is common to any type of line due to the use of the relative units system. Using the described calculation model, a broad parametric analysis of the operating regimes of the circuits with distributed parameters can be made.

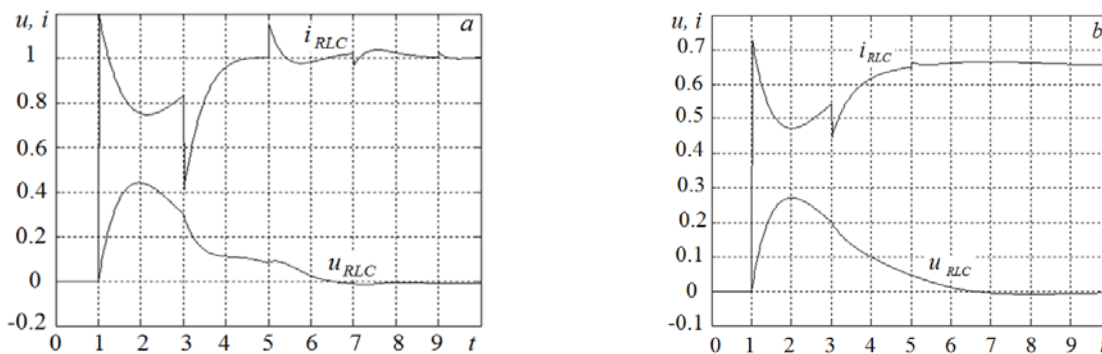


Figure 6. Current and voltage curves in load $R_S = L_S = C_S = 1$ of line in cable with parameters $C = L = a = l = 1$; $R = 1$; $G = 0$ (a) and $C = L = a = l = R = G = 1$ (b).

3.3.5. Modeling a complex regime

The study of the wave processes in the lines with loss indicates a strong influence of the losses in line on the dynamics of the non-stationary process. For this purpose, the calculations of the regime were made in the line $x = l = \lambda / 8$, in which the idle and short-circuiting changes from short circuit to idle regime. These changes are made in the uninterrupted numerical calculation process time interval equal with duration the 5 wave propagations through the cable with distributed parameters. The rate of loss in cable is defined by the dissipation coefficient $\gamma_L = R / L$ and $\gamma_C = G / C$. In the event that $\gamma_L = \gamma_C$ we have the circuit without the distortion of the signal shape, which travels through the distributed parameter circuit. In relative units for the line without distortion the condition $\gamma_L = \gamma_C = 1$ is fulfilled.

The results of the numerical calculation of the regime by changing the load from idle to short regime of the line are shown in "Figure 7" and "Figure 8". The interconnection line between source and load has an influence on process dynamics in this circuit. In order to quantify the degree of influence, some circuit regimes were studied for different line parameter values - from the lossless line (the ideal line) to the line with increased energy losses. As characteristic regimes the electric power transmission regimes were selected through the loss-free line ($\gamma = 0$) the non-distortion line of the electric signals ($\gamma = 1$) and the with distortion line of the electric signal ($\gamma \neq 1$). Depending on the ratio of the transmission line parameters, the essential features of the processes in these circuits are observed. It is essential the energy accumulation regime with increasing the voltage in the lossless line (ideal line), which operates in a short-circuit regime. In the event that no action is taken, the increase in idling voltage can be completed with the electrical breakthrough of the insulation, thus with the complete refusal of the line.

These simulations confirm the robustness of the uninterrupted computational method and its applicability for studying the particularities of the circuits with distributed parameter and concentrated parameters. These numerical simulations have a high degree of precision of the solutions obtained by the finite difference method. At the same time, the solutions are correct and have a high degree of credibility and correspond to the physical essence of the dynamic processes occurring in the circuits with distributed and lumped parameters, included for the detection of certain regimes, which is difficult to notice, even if the analytical solution is known.

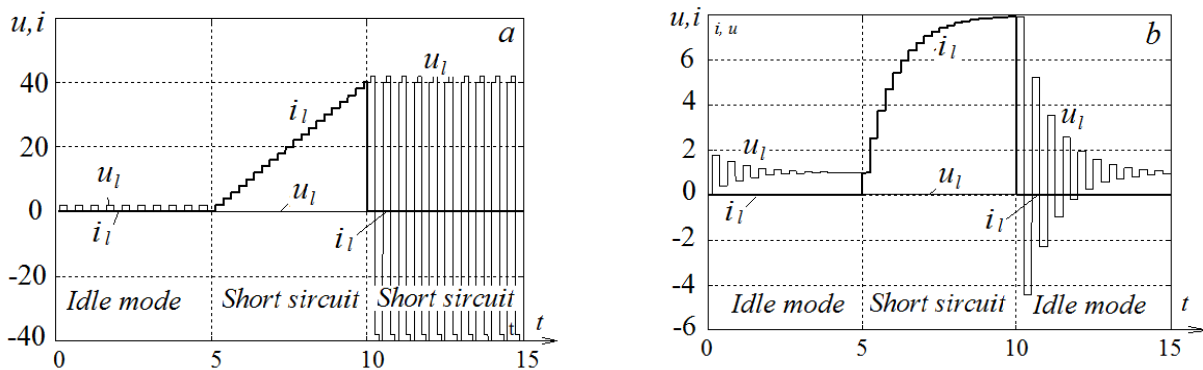


Figure 7. The currents and voltages (curves i_l and u_l) for lines with linear parameters: $\gamma = 0$ (a); $\gamma = 1$ (b), when conditions IM-SCM-IM are met at the line end.

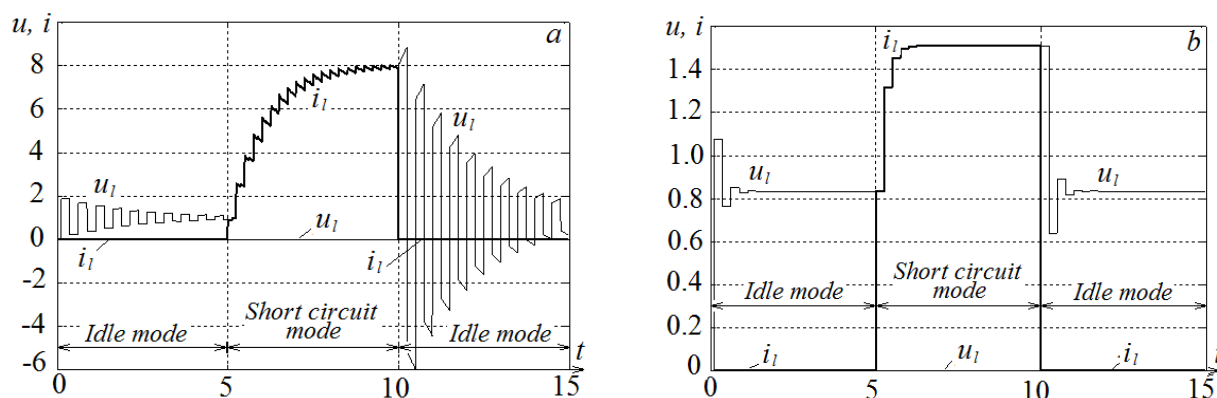


Figure 8. Currents (curve i_1) and voltages (curves u_1) for line with linear parameters: $R=1$, $G=0$, (a) and $\gamma = 5$ (b), when the regime in the lines with the long $x=l=\lambda/8$ it changes in the order: idle - short circuit - idle modes.

Conclusions

The mathematical models of the circuits with distributed and defiled parameters are recommended to be elaborated, using hyperbolic equations with partial derivatives. In this case, equations with partial derivatives can be transformed into differential equations of an independent variable. The use of relative units system for the description of the coefficients and the independent variables allows us to generalize the studied problem and to obtain correct solutions for the electrical lines of different lengths, which ensures the possibility of comparing the technical indices and the efficiency of different circuits.

The process of forming the mathematical model of calculation the regimes in complex structure circuits based on the telegraph equations and procedure to define the initial and boundary conditions in the relative units system were proposed and discussed. In order to obtain numerical solutions, taking into account the peculiarity of the electrical circuit topology and the physical processes occurring over time, it is recommended to perform the mathematical models of simulation of these processes using the finite difference method. The numerical solutions obtained correctly reflect the physics of the processes in these circuits. The finite difference method ensures a high degree of precision of numerical solutions.

Numerical simulations of circuit operating regimes with distributed and lumped parameters allow for the detection of effects that are not visible, even if the analytical solution is known. Thus, it has been found that in the complex circuit consisting of the part with distributed parameters (long line) and the load with lumped parameters (inductance), it is possible to continuously inject the energy into this inductance even from the DC power supply. This phenomenon is a promising research in the future as it can provide a solution for routing with the energy storage process in inductance with superconductivity.

The mathematical models, the proposed design process of these models, have a good degree of flexibility and are suitable for carrying out parametric researches in order to optimize the operation regimes and to ensure the protection and reliability of the circuits in dynamic regimes.

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