Two-Photon Coherent Fields and its Application in Communication

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***Abstract* – It is examined the coherence properties between the Stokes and anti-Stokes fields and its application in Communication. It is proposed novel two-photon entangled sources which take into account the coherence and collective phenomena between these fields. The quantum propriety of realistic sources of powerful coherent bi-boson radiation (coherent entanglement of Stokes and anti- Stokes photons) is analyzed. The possibility of experimental applications of coherence between the Stokes and anti-Stokes photons in quantum communications and cryptography is proposed.**

***Index Terms* – Coherence, correlation function, generation, non-liner optics, quantum-computing.**

I. INTRODUCTION

Quantum Information Science is an emerging field with the potential for revolutionary advances in fields of science and engineering involving computation, communication, precision measurement, and fundamental quantum science.

The property of entanglement between the emitted photons in the processes of light generation has a great impact towards applications dealing with quantum computing, and information security (e.g. [1]). The modern investigations connected with the manipulation of quantum fluctuations of the generated light and there application in transmission and detection of information with high degree of security play an important role in the modern defense problems. The problem of quantum fluctuations and the generation of non-classical electromagnetic fields in multi-photon processes have been the subject of extensive theoretical and experimental studies in recent years, more specifically two-photon coherent generation of light has led to many experimental and theoretical studies recently An interesting behavior of Stokes (e.g. [2]) and anti- Stokes generated modes in the Raman processes (e.g. [3]) can be observed for the small number of the pumped photon in nonlinear media.

In this article it is proposed the new type of generation of coherence states of biboson field and its application in comunication, biophotonics. This field can be regarded as two coherent fields generate in the lasing process of Stokes and anti-Stokes modes in nonlinear medium. Such a quantum correlations can be obtained in the processes of Raman scattering (e.g. [4]). In this article it is obtained new coherent states between Stokes and anti-Stokes fields and it is proposed to use such a fields in quantum communication, holography and biotehnology. It is demonstrated that these collective scattering phenomena take place due to information transfer between the photons of two cavity modes. Studying the quantum fluctuations of the number of photon the new proprieties of the Stokes and Anti-Stokes fields have been found and proposed for communication with hair degree of security of information. The new peculiarities of second order correlation functions between these fields are considered as potential algorithms in cryptography and are expressed through the lasing parameters of the source (e.g. [5]).

This manuscript reports the review of the transmission of information through entangled photons, obtained in parametric down conversion (e.g. [6, 7]). In this articles it is proposed the new method of transmission of information taking in to account the two-field coherent states. The main difference between Ekert model (e.g. [8]) and new possibilities which include the second order coherent effects is given. It is considered that such coherence between the photon of two fields can be conserved in the process of propagation bimodal field through different fibers over long distances after the focusing. The information encapsulated into coherent bi-boson light can be destroyed in the dispersive medium and restored over a certain distance. In both methods of transmission bimodal coherent field, the nature of the quantum communication between two points *A* (Alice) and *B* (Bob) does not allow eavesdropper, *E* (Eve), to know the transferred information. Below we shall discuss the cryptographic aspects bvon the basis of two entanglement photons obtained in parametrical down conversion and our model based on the two-field coherence effect between the Stokes and anti-Stokes fields. These models are described in the section A and B.

II. PROTOCOLS WITH ENTANGLEMENT EFFECT BETWEEN PHOTONS

The transmission of information through entangled photons is based on he the Ekert's protocol (e.g. [8]). This protocol is encoded by its physical nature. The system based on the entanglement effect between the photons consists in the following: Alice receives one of the photons of the pair and Bob the second. Alice and Bob have de same detection basis and for every particle pair everyone choose independently an accidentally axis and measure the polarization along the axis. After that a series of photons pair are transmitted, they announce what axis of polarizer were chosen in the process of measurement and analyze in which cases they obtained the particles simultaneously (e.g. [8]). So, the channel is established. This system is encoded automatically. A detailed description can be found in the works (e.g. [8]). The pulses formed from pairs of entangled photons can be applied in quantum communication and cryptography using the great investigations of quantum optics. It was demonstrated that in the process of transmission of correlated two photons which were obtained with parameter down-conversion effect through two optical fibers, the correlation between the photons pairs is conserved at a very big distance (30 km), and for more than 6 km in free-space (e.g. [9]). For the distance that is mach than 20 km in free space Chinese physicist realizes (e.g. [9]) it today. He thinks that simultaneously with an output to geostationary satellites the communication through quantum cryptography will be possible for distance around 10 thousands km. In other words, the humanity will have cryptographic channels that cannot be listened by eavesdropper (Eve), because the nature of the communication through pairs of photons does not allow this. Below we give the scheme that describes this process of transmission at the distance of 13 km (e.g. [9]). The generator of pairs of photons (probably the nonlinear crystal without a inversion centre (MgO:LiNbO3) is situated in Chinese place Dashu. The flux was expanded using a optic telescope. The signal was compressed with telescopes of the same type at the detectors Alice and Bob situated in USTC (University of Science and Technology of China) and the place Taouhua. The protocols Alice and Bob coincided, that means a high efficiency in the process of transmission of the information.

III. NEW ARHITECTURE USING THE COHERENCE BETWEEN STOKES AND ANTI-STOKES FIELDS

It propose a novel architecture for quantum communication. The amplitude of a simple block of coherent Stokes and anti-Stokes photons obtained after two-photon interaction in scattering lasing effects can be described by the square value of electrical vector $\Pi(t) = g_S g_A \hat{b}^+ \hat{a} Exp[\omega_0 t - (k_a - k_s)z + \varphi_0]$. $\Pi(t) = E_A^+ E_S$,· representing these amplitudes through $E_A^+ = \hat{b}^+ \exp[i(\omega_A t - k_z z)] g_A$ and $E_S^- = \hat{a} \exp[-i(\omega_S t - k_S z)] g_S$, where $\hat{b}^+$ and $\hat{a}^+$ are creation photon operators in the anti-Stokes and Stokes fields respectively, $\hat{b}$ and $\hat{a}$ are annihilation photon operators in these fields. In the quasi-classical limits the amplitude $\Pi_0(t) = g_S g_A \langle \hat{b}^+ \hat{a} \rangle$, has the same proprieties as amplitude of coherent laser field. In this approximation a nice idea is to use the classical of two wave modulation of this square amplitude for transmission of information. At first glance, it is observed that this method does not have a substantial differences in comparison with classical methods of information processing, but if we send this information in dispersive media, which separates anti-Stokes and Stokes photons from coherent entanglement fields, the information is drastically destroyed, because $\langle \hat{b}^+ \rangle$ and $\langle \hat{a} \rangle$ take zero values. The possibility of restoration of information on the square amplitudes $\Pi_0(t)$, is interesting problem of many particle coherent states, formed from blocks of Stokes and anti-Stokes photons. These studies are necessary because in a bi-boson lasing effect (e.g. [10]), the photon statistics depend on the statistics of the ignition field (e.g. [11]). Of course, the start up from vacuum fluctuations preserves the entanglement character of the generated Stokes and anti-Stokes coherence state. This effect is very interesting in quantum communication.

This manuscript propose an interesting effect that takes into account the classical method of registration of information. As Π(t) plays the role of electromagnetic field intensity strength for the two-fields Stokes and anti-Stokes, at the detector can be considered as a classical field described by $\Pi(t) = \Pi_0(t)\cos[\tilde{\omega}t - (k_a - k_s)z + \varphi_0]$, where Π_0 is the envelop of cooperative two-photon interaction in scattering processes. A large number of modes in the coherent states give as the possibilities of the increase the security of information storages in bimodal field (e.g. [9-11]). In this approximation, the classical information may be introduced in the amplitude $\Pi_0(t)$. Such registration of information may have nothing to do with the traditional method. If the bi-boson pulses pass through a dispersive medium, the anti-Stokes and Stokes photons from the field change their directions. Focusing the anti-Stokes and Stokes photons into different optical fibers we are totally dropping the coherence among the photons. However, after a certain time interval, anti-Stokes and Stokes photons from the field are mixed again, and we can observe that the coherence is restored (see Fig. 1).

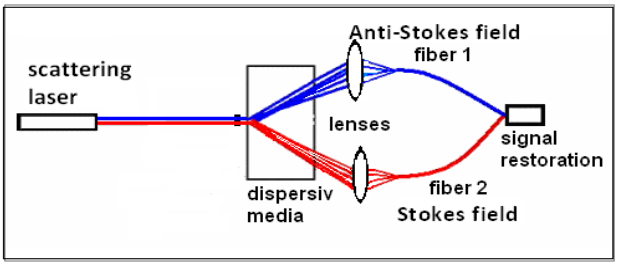

Fig. 1.

The coherent state obtained in two-photon coherent emission $|\Psi\rangle = \exp[\Pi^+ - \Pi^-]|0\rangle$, takes into account not only entanglement between the pair photons but the coherence between the fields too, and can be used in mixed processing problems in which the quantum entanglement between the Stokes and anti-Stokes photons is used simultaneously with classical coherence (e.g. [6]) between the fields. Of course the probability of experimental realization of logic gates for quantum circuits in this case increase.

The coherent states between Stokes and anti-Stokes fields can be realized in cooperative scattering effects (e.g. [11]). In order to obtain the cooperative generation of the coherences states between Stokes and anti-Stokes fields we consider a stream of atoms (e.g. [10,14,15]) with two levels travelling through the cavity. The total Hamiltonian which describes the interaction of the atoms with Stokes and anti-Stokes fields of the cavity can be represented through the atomic and field operators

$$H_T = \sum_{j=1}^{N} \hbar\omega_0 R_{zj} + \hbar\omega_a a^+ a + \hbar\omega_b b^+ b + i\sum_{j=1}^{N} G(k_a, k_b)\left\{\left\langle R_j^-(t) J^+ - J^- R_j^+(t)\right\rangle\right\} \quad (1)$$

where the last term represent the interaction Hamiltonian. Here R_{zj} is the population inversion of atom j; $\hat{R}_j^+$ and $\hat{R}_j^-$

represent the operators which describe the transitions from $|g\rangle$- ground state to $|e\rangle$- excited state and from $|e\rangle$- excited state to $|g\rangle$- ground state, respectively (e.g. [10]). The operator $a^+(a)$ is the creation (annihilation) of Stokes photons and $b^+(b)$ is the creation (annihilation) of anti-Stokes field operators. The interaction constant $G(k_a,k_b)$ describes the effective nonlinear coupling of atom *j* with cavity modes k_a and k_b with the energies $\hbar\omega_a$ and $\hbar\omega_b$. In order to describe the scattering processes, let us introduce the collective operators for Stokes and anti-Stokes modes, $J^+ = ab^+$ and $J^- = ba^+$ (e.g. [12]). The operator $J^+ = ab^+$ describes the simultaneously process of creation of anti-Stokes and annihilation of Stokes photons. The inverse process is described by operator: $J^- = ba^+$. As the full number of photons in the cavity, in the small time moment the interaction is conserved, we shall introduce the operator of photons inversion between the Stokes and anti-Stokes photons: $J_z = (b^+b - a^+a)/2$ and energy difference $\hbar\tilde{\omega} = \hbar\omega_b - \hbar\omega_a$.

Taking into account, that the lifetime of atoms in the cavity are shorter than the time of scattering processes and considering that the atomic system is prepared in excited state, let us eliminate the atomic operators $\hat{R}_j^+$ and $\hat{R}_j^-$ from Heisenberg equation of arbitrary field operator. By representing the operators $\hat{R}_j^+$ and $\hat{R}_j^-$ through the field operators $\hat{J}^+$ and $\hat{J}^-$ in according with Hamiltonian (1). After consecutive elimination of free parts of these operators the master equation for Stokes and anti-Stokes field can be represented in the forth order on the interaction constant $G_j(k_a,k_b)$

$$\frac{d}{dt}J^+(t) = -2\tilde{\alpha}_1\langle J_z(t)J^+(t)\rangle + 4\tilde{\alpha}_2\langle J_zJ^+J^-J^+\rangle, \quad \frac{d}{dt}\langle J^-(t)\rangle = \left[\frac{d}{dt}\langle J^+(t)\rangle\right]^+ \qquad (2)$$

The behaviour over time of the mean value of operators $J^+(t)$ and $J^-(t)$ can be found in accordance with the generalized equation propose in paper (e.g. [12]). In semi classical approximation, when the fluctuations of these operators are neglected $J^{\pm}(t) \approx <\hat{J}^{\pm}(t)>$ and $J_z(t) \approx <\hat{J}_z(t)>$, the equations (2) for mean values of these operators $\hat{J}^+$ and $\hat{J}^-$ take the folowing simple expression

$$\frac{d}{dt}\langle J^+(t)\rangle = -2\tilde{\alpha}_1\langle J_z\rangle\langle J^+(t)\rangle + 4\tilde{\alpha}_2\langle J_z\rangle\langle J^+\rangle\langle J^-\rangle\langle J^+\rangle. \qquad (3)$$

Considering that, at initial stage of ignition of generation the inversion operator can be aproximate with $J_z \approx -j$, and following the idea proposed in paper (e.g. [11]), the equation (3) can be represented by the generalized potential function, $V(J^+,J^-)$, $d<J^+(t)>/dt = \partial V/\partial <J^->$. In accordance with this definition, we obtain the following potential function

$$V(z) = -2\tilde{\alpha}_1 j|z|^2 + 2\tilde{\alpha}_2 j|z|^4 \qquad (4)$$

with the minimum in point $|z|_{\min} = \tilde{\alpha}_1/(2\tilde{\alpha}_2)$, where $|z| = |J^+| = |J^-|$. The dependence of this potential V(z) as function of the amplitude $|z|$ decrease achieving $z_{\min}$ and after that increase. As follows from the expression (4), the amplitude value of two-photon coherent fields $|z|$ is proportional with the ratio between scattering rate and diffusion coefficient $\tilde{\alpha}_2$ and increases with increasing of scattering rate, $\tilde{\alpha}_1$, This steady state solution describes the stabilization process in the resonator.

We are interested in the behavior of quantum fluctuations of this bi-field intensity, $\hat{J}^+(t)\hat{J}^-(t)$, in the process of time evolution to steady state: $\Delta^2 = G_2(t) - G_1^2(t)$ where $G_1(t) = <\hat{J}^+(t)\hat{J}^-(t)>$ and $G_2(t) = <\hat{J}^+(t)\hat{J}^+(t)\hat{J}^-(t)\hat{J}^-(t)>$, are the intensity and square of intensity of bi-boson field consisted from Stokes and anti-Stokes fields. Following the method proposed in paper (e.g. [11]) let now found the behavior of correlation functions $G_1(t)$ and $G_2(t)$. Taking in to account the solution of quantum equations (2), in the Fig. 2 it is plotted the dependece of correlation function $G_1(t)$ on the relative time t/(2α₁) for the following relative expression for parameters α_1 =0.1, α_2 =0.01 and $\alpha_1/2\alpha_2$ =5. This plot (see Fig. 2) demonstrated the good stabilization of second order coherence between the Stokes and anti- Stokes photons.

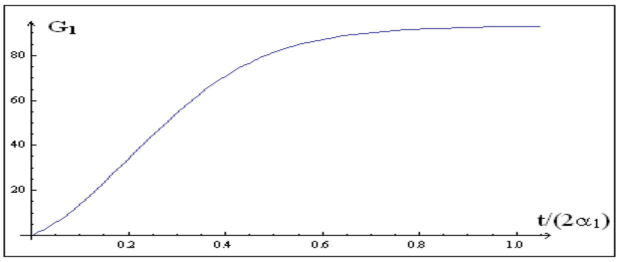

Fig. 2 The dependece of correlation function $G_1(t)$ on the relative time t/(2α₁) for the following value of parameters α_1 =0.1, α_2 =0.01 and $\alpha_1/2\alpha_2$ =5.

As follows from the Fig. 2 with increasing of time moment the correlation function achieved the maximal value, which corresponds to steady state stabilization. In Fig. 3 we have plotted the time dependence of square fluctuations Δ^2. It is observed the time decreasing of quantum fluctuations of coherent amplitudes proposed in communication with Stokes and anti-Stokes photons. As follows from the Fig. 3, with the stabilization of lasing process the square fluctuation becomes negative.

IV. CONCLUSION

The recent advances in quantum communication by using quantum optical proprieties of light are reviewed. The quantum communication protocols are carefully discussed. On the basis of these protocols many laboratories work for development and implementation of quantum optics devices. It is proposed a new method of quantum communication, which takes in to account the coherence between the entangled photon fields and the application of this effect in quantum communication. Considering coherent and corpuscular properties of light, consisted of photon fields, the new scheme for quantum communication has been offered for the quantum communications.

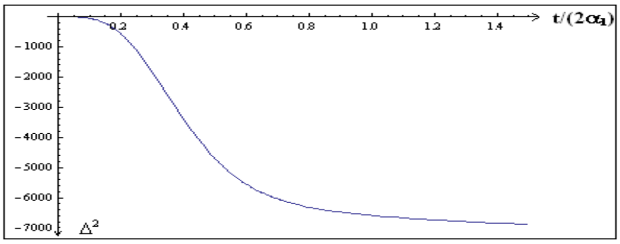

Fig. 3 The dependences of square fluctuations, Δ^2, as function of relative time $t/(2\alpha_1)$, for the same parameter as in Fig. 2.

This effects have many analogies with anti-bunging behavior of Stokes and anti-Stokes fields, described by $\hat{J}^+$ and $\hat{J}^-$ operators of SU (2) group (e.g. [11,16]).

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