Strong coupling diagrammatic approach to correlated polarons

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Abstract — **The correlation function of phonon clouds of correlated polarons is calculated in second and fourth order of perturbations series and the origin of renormalization perturbation series is established. Diagram representations of different contributions are given.**
***Index Terms* — Anderson-Holstein model, correlated polarons, diagrammatic technique, polarization operator , Wick's theorem.**

I. INTRODUCTION

In papers [1, 2] we discussed in simplest approximation the dynamics of phonon clouds which belonged to strongly correlated polarons. This investigation supposed that the system is in normal state and the structure of phonon clouds is simplest. Now we shall admit the presence of anomalous propagators [3] and the situation when temperature of the system is less the critical value of superconducting transition. The basic equation which determines the phonon clouds dynamic has the form:

$$\Phi(\tau,\tau') = \left\langle T e^{ig(P(t)-P(t'))} U(\beta) \right\rangle, \quad (1)$$

where $U(\beta)$ - evolution operator in the interaction representation of the Anderson-Holstein model

$$U(\beta) = T\exp\left(-\int_0^\beta d\tau H_{\text{int}}(\tau) d\tau\right), \quad (2)$$

$$H_{\text{int}}(\tau) = \sum_\sigma \left(\bar{\bar{f}}_\sigma(\tau) b_\sigma(\tau) e^{-igP(\tau)} + \bar{b}_\sigma(\tau) f_\sigma(\tau) e^{igP(\tau)} \right), \quad (3)$$

f_σ^+, f_σ are operators of impurity electrons, b_σ^+, b_σ are operators of conduction electrons in localized version

$$b_\sigma = \frac{1}{\sqrt{N}} \sum_R V_{k\sigma} C_{k\sigma}. \quad (4)$$

$V_{k\sigma}$ is matrix element of hybridization, g is constant of electron-phonon interaction and p is momentum of phonons. T is the operator of chronological ordering.

II. PERTURBATION SERIES

In the second order of perturbation theory we have

$$\Phi^{(2)}(\tau/\tau') = \frac{1}{2}\Big\langle T e^{igP(\tau)-igP(\tau')} \int_0^\beta\int_0^\beta d\tau_1 d\tau_2 \sum_{\sigma_1}$$
$$\sum_{\sigma_2}\left[\bar{f}_{\sigma_1}(\tau_1) b_{\sigma_1}(\tau_1) e^{-igP(\tau_1)} + \bar{b}_{\sigma_1}(\tau_1) f_{\sigma_1}(\tau_1) e^{igP(\tau_1)} \right] \times \quad (5)$$
$$\times\left[\bar{f}_{\sigma_2}(\tau_2) b_{\sigma_2}(\tau_2) e^{-igP(\tau_2)} + \bar{b}_{\sigma_2}(\tau_2) f_{\sigma_2}(\tau_2) e^{igP(\tau_2)} \right]\Big\rangle =$$
$$= -\int_0^\beta\int_0^\beta d\tau_1 d\tau_2 \sum_{\sigma_1\sigma_2} \Big\{ \frac{1}{2} \Phi_0(\tau\tau_1\tau_2 \mid \tau')$$
$$\left\langle T f_{\sigma_1}(\tau_1) f_{\sigma_2}(\tau_2) \right\rangle \left\langle T \bar{b}_{\sigma_1}(\tau_1) \bar{b}_{\sigma_2}(\tau_2) \right\rangle \Big\},$$

where

$$\Phi_0(\tau_1\tau_2 \mid \tau_3\tau_4) = \left\langle T e^{igP(\tau_1)+igP(\tau_2)-igP(\tau_3)-igP(\tau_4)} \right\rangle =$$
$$= \exp\left\{ -\frac{g^2}{2} \left\langle T\left(P(\tau_1)+P(\tau_2)-P(\tau_3)-P(\tau_4)\right)^2 \right\rangle \right\} \quad (6)$$
$$= \exp\{\sigma(|\tau_1-\tau_3| + \sigma|\tau_1-\tau_4| + \sigma(|\tau_2-\tau_3| +$$
$$+\sigma|\tau_2-\tau_4| - \sigma(|\tau_1-\tau_2| - \sigma|\tau_3-\tau_4| - 2\sigma(\beta))\},$$

$$\sigma(|\tau_1-\tau_2|) = g^2 \left\langle T P(\tau_1) P(\tau_2) \right\rangle. \quad (7)$$

The non-diagonal quantities $\Phi_0(\tau \mid \tau_1\tau_2\tau')$ and $\Phi_0(\tau\tau_1\tau_2 \mid \tau')$ are omitted as small quantities. In such a way we have more simple equation:

$$\Phi^{(2)}(\tau\tau') \approx -\int_0^\beta\int_0^\beta d\tau_1 d\tau_2 \sum_{\sigma_1\sigma_2} \Phi_0(\tau\tau_1 \mid \tau_2\tau')$$
$$\left\langle T f_{\sigma_1}(\tau_1) \bar{f}_{\sigma_2}(\tau_2) \right\rangle \left\langle T b_{\sigma_2}(\tau_2) \bar{b}_{\sigma_1}(\tau_1) \right\rangle \quad (8)$$

Now we shall use Wick's theorem of phonon clouds formulated in paper [4]

$$\Phi_0(\tau\tau_1 \mid \tau_2\tau') = \Phi_0(\tau \mid \tau_2)\Phi_0(\tau_1 \mid \tau') +$$
$$+\Phi_0(\tau \mid \tau')\Phi_0(\tau_1 \mid \tau_2) \quad (9)$$

After omitting the disconnected part of diagram we obtain

$$\Phi^{(2)}(\tau\tau') = \int_0^\beta\int_0^\beta \Phi_0(\tau\tau_2 \Pi^{(2)}_{\sigma_2\sigma_1}(\tau_2 \mid \tau_1)\Phi_0(\tau_1\tau')) d\tau_1 d\tau_2, \quad (10)$$

where the polarization operator $\Pi^{(2)}$ of second order has the form

$$\Pi^{(2)}_{\sigma_2\sigma_1}(\tau_2 \mid \tau_1) = -\left\langle T f_{\sigma_1}(\tau_1) \bar{f}_{\sigma_2}(\tau_2) \right\rangle \left\langle T b_{\sigma_2}(\sigma_2) \bar{b}_{\sigma_1}(\tau_1) \right\rangle. \quad (11)$$

Now we shall discuss the contributions of the forth order of the correlation functions

$$\Phi^{(4)}(\tau\tau') = \frac{1}{4!}\left\langle e^{igP(\tau)-igP(\tau')}\int\ldots\int d\tau_1\ldots d\tau_4 H_{\text{int}}(\tau_1)\ldots H_{\text{int}}(\tau_4)\right\rangle,$$

After preserving the main contributions we obtain

$$\Phi^{(4)}(\tau\tau') = \frac{1}{4}\int\ldots\int d\tau_1\ldots d\tau_4\left(\Phi_0(\tau\tau_1\tau_2\,|\,\tau_3\tau_4\tau')\times\right. \\ \left.\times\left\langle Tf_1 f_2 \overline{f}_3 \overline{f}_4\right\rangle\left\langle T\overline{b}_1\overline{b}_2 b_3 b_4\right\rangle\right). \quad (12)$$

The result of such correlations is depicted on the Fig. 1. The first diagrams of Fig. 1 are the second order contribution. This bubble contains two propagator lines from conduction electrons and one from localized electrons.

The all other diagrams of this figure are the result of renormalization of the simplest bubble.

There are four different possibilities of renormalization. The first possibility is the increasing the number of the bubbles which is demonstrated by the second and third diagrams with two bubbles. The second diagrams of the first row are the contribution of normal state and the next diagram is the contribution of superconducting phase.

There are two possibilities of renormalization determined by increasing the structure of lines and vertices of bubbles and the last possibilities is determined by irreducible contributions which are depicted by the last row of Fig. 1.

The first contribution was analyzed in details in paper [1].

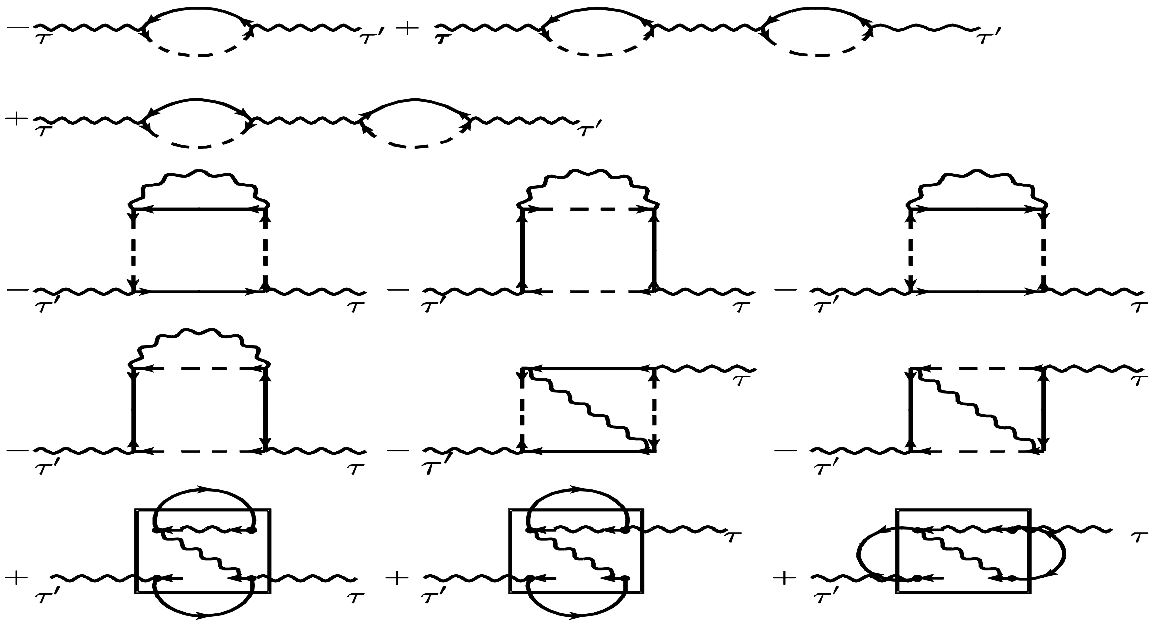

Fig.1 Diagrams of second and fourth order of perturbation theory for $\Phi(\tau,\tau')$.

III. CONCLUSION

By using the generalized Wick theorem for correlated impurity electrons, ordinary Wick's theorem of conducting non-correlated electrons and special Wick's theorem for phonon clouds formulated in paper [4], the values of phonon correlation function of second and fourth order of perturbation series are obtained and different renormalization processes are analyzed.

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