# PROPAGATION OF LASER RADIATION IN NONLINEAR COUPLER IN THE CASE OF TWO-PHOTON EXCITATION OF BIEXCITONS 

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#### Abstract

The propagation peculiarities of coherent laser radiation in two identical semiconductor waveguides are investigated. The system of the nonlinear differential equations is derived, which describes the intensity of propagating pulses. The periodic, quasiperiodic and chaotic energy transfer is predicted, considering the pulse portraits on two connected Poincare spheres.


The propagation features and the interaction of laser radiation in two identical parallel semiconductor waveguides are considered. Waveguides form a nonlinear optical coupler. We suppose that two pulses of laser radiation with the frequencies $\omega_{1}$ and $\omega_{2}$ and the field amplitudes of the pulse envelopes $E_{1}$ and $E_{2}$ incident on the end face of one of the waveguides (Fig. 1). The sum of the frequency of both pulses is in the exact resonance with the self-frequency of the biexciton state in the medium of waveguides (Fig. 1). Propagating along the waveguides the photons of these pulses in pairs excite the biexcitons with which they interact. Due to the tunnel effect the energy of propagating pulses is transferred periodically from one waveguide to another and vise versa. The main problem is the determination of the spatial distribution of the fields of both pulses in the waveguides and the possibility of the realization of the nonlinear self-switching phenomenon.

In the frame of steady-state and of the slowly varying in space envelopes approximations we have obtained the system of nonlinear equations for the fields $f_{1}$ and $f_{2}$ in the first guide and $g_{1}$ and $g_{2}$ for the fields in the second guide of the coupler:

$$
\begin{align*}
& d f_{1} / d x=-i \alpha I_{2} f_{1}+i \gamma g_{1},  \tag{1}\\
& d f_{2} / d x=-i \alpha I_{1} f_{2}+i \gamma g_{2}, \tag{2}
\end{align*}
$$

$$
\begin{align*}
& d g_{1} / d x=-i \alpha J_{2} g_{1}+i \gamma f_{1}  \tag{3}\\
& d g_{2} / d x=-i \alpha J_{1} g_{2}+i \gamma f_{2} \tag{4}
\end{align*}
$$

where $x$ is the coordinate, $\alpha$ is the constant of two-photon biexciton excitation and $\gamma$ is the linear coupling constant and

$$
\begin{equation*}
I_{1}=\left|f_{1}\right|^{2}, I_{2}=\left|f_{2}\right|^{2}, \quad J_{1}=\left|g_{1}\right|^{2}, \quad J_{2}=\left|g_{2}\right|^{2} \tag{5}
\end{equation*}
$$

are the intensities.
We can form the components of polarizations

$$
\begin{align*}
& Q_{1}=i\left(g_{1}^{*} f_{1}-g_{1} f_{1}^{*}\right), Q_{2}=i\left(g_{2}^{*} f_{2}-g_{2} f_{2}^{*}\right), \\
& R_{1}=g_{1}^{*} f_{1}+g_{1} f_{1}^{*}, R_{2}=g_{2}^{*} f_{2}+g_{2} f_{2}^{*}, \tag{6}
\end{align*}
$$

and we obtain the set of nonlinear equations for the intensities

$$
\begin{align*}
& I_{1}^{\prime}=-\gamma Q_{1}, I_{1}^{\prime}=-\gamma Q_{1}, J_{1}^{\prime}=\gamma Q_{1}, J_{2}^{\prime}=\gamma Q_{2},  \tag{7}\\
& Q_{1}^{\prime}=\alpha\left(I_{2}-J_{2}\right) R_{1}+2 \gamma\left(I_{1}-J_{1}\right),  \tag{8}\\
& Q_{2}^{\prime}=\alpha\left(I_{1}-J_{1}\right) R_{2}+2 \gamma\left(I_{2}-J_{2}\right),  \tag{9}\\
& R_{1}^{\prime}=-\alpha\left(I_{2}-J_{2}\right) Q_{1},  \tag{10}\\
& R_{2}^{\prime}=-\alpha\left(I_{1}-J_{1}\right) Q_{2}, \tag{11}
\end{align*}
$$

where the prime at the variables is the derivative on $x$.
From (7)-(11) we obtained the following integrals of motion

$$
\begin{align*}
& I_{1}+J_{1}=I_{10}, \quad I_{2}+J_{2}=I_{20},  \tag{12}\\
& Q_{1}^{2}+R_{1}^{2}=4 I_{1} J_{1}, \quad Q_{2}^{2}+R_{2}^{2}=4 I_{2} J_{2},  \tag{13}\\
& R_{1}+R_{2}+\alpha / \gamma\left(I_{1} J_{2}+I_{2} J_{1}\right)=0, \tag{14}
\end{align*}
$$

where $I_{10}$ and $I_{20}$ are the intensities of two waves incident of the front endfase of the first waveguide, i.e.

$$
\begin{equation*}
I_{1 \mid x=0}=I_{10}, I_{2 \mid x=0}=I_{20}, J_{1 \mid x=0}=J_{2 \mid x=0}=0 . \tag{15}
\end{equation*}
$$

Introducing new normalized variables

$$
\begin{array}{ll}
x_{1}=\alpha_{1}\left(1-2 \frac{J_{1}}{I_{10}}\right), & x_{2}=\alpha_{2}\left(1-2 \frac{J_{2}}{I_{20}}\right), \\
y_{1}=\alpha_{1} \frac{Q_{1}}{I_{10}}, & y_{2}=\alpha_{2} \frac{Q_{2}}{I_{20}},  \tag{16}\\
z_{1}=\alpha_{1} \frac{R_{1}}{I_{10}}, & z_{2}=\alpha_{2} \frac{R_{2}}{I_{20}},
\end{array}
$$

where $\alpha_{1}=\frac{\alpha I_{10}}{2 \gamma}, \alpha_{2}=\frac{\alpha I_{20}}{2 \gamma}$, we have obtained the system of six nonlinear equations for new variables

$$
\begin{array}{ll}
x_{1}^{\prime}=-y_{1}, & x_{2}^{\prime}=-y_{2}, \\
y_{1}^{\prime}=x_{1}+z_{1} x_{2}, & y_{2}^{\prime}=x_{2}+z_{2} x_{1},  \tag{17}\\
z_{1}^{\prime}=-y_{1} x_{2}, & z_{2}^{\prime}=-y_{2} x_{1}
\end{array}
$$

together with three integrals of motion

$$
\begin{align*}
& x_{1}^{2}+y_{1}^{2}+z_{1}^{2}=\alpha_{1}^{2} \\
& x_{2}^{2}+y_{2}^{2}+z_{2}^{2}=\alpha_{2}^{2}  \tag{18}\\
& x_{1} x_{2}-z_{1}-z_{2}=\alpha_{1} \alpha_{2}
\end{align*}
$$

where the only normalized parameters of our theory are $\alpha_{1}$ and $\alpha_{2}$. From (18) we can see that that the evolution of the system is represented as the bound motion of two displayed points on the surfaces of two spheres enclosed each other, the radii of which $\alpha_{1}$ and $\alpha_{2}$ are determined by the initial conditions. We obtained the exact analytical solutions of the system of equations only for the case when the radii of the spheres are equal $\alpha_{1}=\alpha_{2}=\alpha$. At the high level of excitations of the system from the end face of one of waveguides it takes place the periodic complete energy transfer of the propagating pulses and the connection length (the minimal distance on which the maximal energy transfer takes place) slowly increases with the increase of the pump intensity. In Fig. 2 we presented the solutions (spatial variation of variables) $x_{1}=x_{2}=x, y_{1}=y_{2}=y$ and $z_{1}=z_{2}=z$ for different values of parameter $\alpha$. We can see, that every variable $x, y$ and $z$ exhibits the periodical spatial change. The period of oscillations depend only on the parameter $\alpha$. The increase of $\alpha$ leads to the increase of spatial period for $0 \leq \alpha \leq \alpha_{c r}=2$ and for $\alpha>2$ the period decreases.

If the level of excitations approaches the critical one the connection length increases rapidly and all energy of pulses is transferred. For the case of critical pump intensity it takes place the aperiodic transferring regime and the half of energy is transferred. At the high level of excitation more than the critical one the connection length rapidly decreases and the part of transferred energy is small. We predicted the self-switching phenomenon, which consists in the strong change of the output in the case of small change of the input intensity. The numerical investigations of the system of equations shows the existence of the complex propagations regimes depending on the parameters, the periodic, quasiperiodic and chaotic solutions, pulling-out and pushing-in of phase trajectories from the definite regions of the phase space.


Fig 1. Diagram of the two-photon quantum transitions a) and scheme of the coupler under investigation $b$ ).


Fig. 2 Spatial variation of variables $x, y$ and $z$.

