ABOUT A MOVEMENT EQUATION GENERATED BY A MECHANIC PROBLEM

Gr. Tara
"George Baritiu" University, Brasov

1. INTRODUCTION

According to the theory in domain, the Lagrange movement equation has the form:

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{q}}\right)+\frac{\partial E_p}{\partial q}=Q\ , \tag{1}$$

where:

q – represent the generalized coordinates
E_c – kinetic energy of the system
E_p– potential energy of the system
Q – generalized forces.

In our case these elements can be determined by measurements generated by instant translations and rotations of the entire motor converter, the translations and rotations of the pistons and piston rods face to unit engine, rotation of crankshaft, all these in view of the calculation if kinetic energy.

The potential energy due to the vibrations is generated by forces and elastic moments of the fixing bearing of the engine converter on the chassis of an automobile.

The generalized forces due mainly to the pressure of the gas from the engine cylinder.

These equations transposed for an engine group with elastic bearing have the form:

$$[M]\cdot\{\ddot{q}\}+[C]\cdot\{\dot{q}\}+[K]\cdot\{q\}=\{f\} \tag{2}$$

where:

[M] – inertia matrix
[C] – damping matrix
[K] – rigidity matrix.

2. THE INTEGRATION OF THE MOVEMENT EQUATION BY THE METHOD OF MODAL ANALYSIS

In our case, because the damping matrix [C] has elements with very low values, it can be practically neglected as opposed to the other terms of the movement equation.

Thus the matrix movement equation of the engine group elastic bearing has the form:

$$[M]\cdot\frac{d^2}{dt^2}\{q\}+[K]\cdot\{q\}=\{f\}. \tag{3}$$

The integration of the equations system is done without many calculations, by the method of modal analysis.

Practically we determine the pulsation proper values of the dynamic system of elastic engine bearing from the equation:

$$\det\left([K]-\omega^2[M]\right)=0\,, \tag{4}$$

noted with $\omega_i\,, i=\overline{1,6}$, where $\omega=2\pi\cdot N$, N being the speed of the engine shaft in rotations/minute ($\omega=d\varphi/dt$ - angular velocity of the engine shaft, because $\varphi=\omega\cdot t$).

Then we determine the latent vectors, from the matrix equations:

$$\begin{gathered}([D]-\omega_i\cdot[I])\{v_i\}=\{0\},\\ i=\overline{1,6}\end{gathered}\ , \tag{5}$$

where:

$$[D]=[M]^{-1}\cdot[K],$$

matrix for which the proper values are in fact calculated, as well as the latent vectors.

For each value of the shaft rotation angle - φ, which takes values between 0 and 720^0, with a pre-established step, the program generates one set of 6 proper values, as well as a set of 6 latent vectors.

With the help of the latent vectors thus determined, we define the modal matrix of the system:

$$[\Phi]=\left[\{v_1\},\{v_2\},\{v_3\},\{v_4\},\{v_5\},\{v_6\}\right], \tag{6}$$

this being in fact a normalization.

For a quick integration the generalized forces vector is $\{f\}$, decomposed in Fourier series:

$$\begin{gathered}[M]\cdot\frac{d^2}{dt^2}\{q\}+[K]\cdot\{q\}=\\ \{f_0\}+\sum_{j=1}^{8}\{f_{cj}\}\cdot\cos(j\cdot\varphi)+\sum_{j=1}^{8}\{f_{sj}\}\cdot\sin(j\cdot\varphi)\end{gathered}\ . \tag{7}$$

Applying the following linear transformation to the generalized coordinates:

$$\{q\}=[\Phi]\cdot\{\xi\}, \quad (8)$$

with the main coordinates vector, the matrix [M] and [K] are diagonal and we obtain the following matrix equation:

$$\left[M^*\right]\cdot\frac{d^2}{dt^2}\{q\}+\left[K^*\right]\cdot\{q\}= \left\{f_0{}^*\right\}+\sum_{j=1}^{8}\left\{f_{cj}{}^*\right\}\cdot\cos(j\cdot\varphi)+\sum_{j=1}^{8}\left\{f_{sj}{}^*\right\}\cdot\sin(j\cdot\varphi), \quad (9)$$

where

$$[M^*]=[\Phi]^T\cdot[M]\cdot[\Phi],$$

$$[K^*]=[\Phi]^T\cdot[K]\cdot[\Phi],$$

$$[f^*]=[\Phi]^T\cdot[f].$$

Its solutions are immediate:

$$\xi_i=\frac{f_{0i}}{m_i^*\cdot\omega_i^2}+\frac{1}{m_i^*}\cdot\sum_{j=1}^{8}\frac{f_{cji}^*}{\omega_i^2-(j\omega)^2}\cdot\cos(j\varphi)+ \frac{1}{m_i^*}\cdot\sum_{j=1}^{8}\frac{f_{sji}^*}{\omega_i^2-(j\omega)^2}\cdot\sin(j\varphi) \quad (10)$$

$$i=\overline{1,6}.$$

Coming back to the transformation we made, we have:

$$\{q\}=\{a_0\}+\sum_{j=1}^{8}\{a_j\}\cdot\cos(j\omega t)+ \sum_{j=1}^{8}\{b_j\}\cdot\sin(j\omega t), \quad (11)$$

meaning the generalized coordinates of the engine elastic bearing system, and the movements of the mass center of the engine converter (x, y, z) and the rotations around the main inertia axis (α,β,γ).

For the determination of the speeds and main accelerations generalized, we use the formulas:

$$\dot{\xi}_i=-\frac{1}{m_i^*}\cdot\sum_{j=1}^{8}\frac{j\omega\cdot f_{cji}^*}{\omega_i^2-(j\omega)^2}\cdot\sin(j\varphi)+ \frac{1}{m_i^*}\cdot\sum_{j=1}^{8}\frac{j\omega\cdot f_{sji}^*}{\omega_i^2-(j\omega)^2}\cdot\cos(j\varphi), \quad (12)$$

$$i=\overline{1,6}$$

$$\ddot{\xi}_i=-\frac{1}{m_i^*}\cdot\sum_{j=1}^{8}\frac{(j\omega)^2\cdot f_{cji}^*}{\omega_i^2-(j\omega)^2}\cdot\cos(j\varphi)- \frac{1}{m_i^*}\cdot\sum_{j=1}^{8}\frac{(j\omega)^2\cdot f_{sji}^*}{\omega_i^2-(j\omega)^2}\cdot\sin(j\varphi) \quad (13)$$

$$i=\overline{1,6}.$$

Coming back to the transformation made will be done with the formulas:

$$\{\dot{q}\}=[\Phi]\cdot\{\dot{\xi}\},$$
$$\{\ddot{q}\}=[\Phi]\cdot\{\ddot{\xi}\}. \quad (14)$$

To make the calculation we considered the first 8 harmonics whose amplitudes being significant, we appreciated that they assure the approximation wanted with the 2^0 step for the angle values φ, the determined result may be graphically represented, each of them containing a generalized coordinate q together with its differentials $\dot{q}$ and $\ddot{q}$.

2. THE ESTABLISHMENT OF THE EFFORT SENT BY THE CHASSIS ENGINE CONVERTER

Then we calculated the specific dynamic solicitations, sent through the elastic bearing to the chassis engine, as a result of the vibrations due to the inertia forces and the gas forces from the time of the engine functioning.

These can be determined by taking into consideration the spatial movements of the fixing points of the elastic bearing of the engine converter, which is determined in function of the generalized coordinates vector $\{q\}$, formed by the three movements of the mass center (x, y, z) and the three independent rotations (α,β,γ) around the fixed coordinate system axis.

Thus, in function of vector $\{q\}$ we can determine the movements of each elastic center $s_i, i=\overline{1,3}$, united with the engine converter, characteristically to the elastic bearing i, with the help of the relation:

$$\begin{Bmatrix}x_{si}\\y_{si}\\z_{si}\end{Bmatrix}=\begin{Bmatrix}x\\y\\z\end{Bmatrix}+\begin{bmatrix}0&-\sin\gamma&\sin\beta\\ \sin\gamma&0&-\sin\alpha\\ -\sin\beta&\sin\alpha&0\end{bmatrix}\cdot\begin{Bmatrix}a_{si}\\b_{si}\\c_{si}\end{Bmatrix}, \quad (15)$$

where a_{si}, b_{si}, c_{si} represents the coordinates of the elastic center, in report with the mobile coordinates system, united with the engine converter.

The forces sent by the engine converter to the chassis can be determined from the relation:

$$\begin{Bmatrix} F_{sxi} \\ F_{syi} \\ F_{szi} \end{Bmatrix} = \begin{bmatrix} k_{xi} & 0 & 0 \\ 0 & k_{yi} & 0 \\ 0 & 0 & k_{zi} \end{bmatrix} \cdot \begin{Bmatrix} x_{si} \\ y_{si} \\ z_{si} \end{Bmatrix}, \qquad (16)$$

where k_{xi}, k_{yi}, k_{zi} represent elastic constants of the analyzed bearing.

For each support correspondent to each generalized coordinates set q, previously calculated and already memorized in a file, according to the formulas mentioned above, we determine the forces sent by the engine to the chassis.

As it results from these formulas, the forces sent by the engine converter to the chassis depend on the special movement of the group (generated by the torque of the inertia forces and gas burning forces), and also on the elastic constants and the fixing elastic elements positions.

Grouped like this on bearings we graphically represent these forces.

3. POSSIBILITIES AND PERSPECTIVES

With the substitutes $\{\dot{q}\} = \{u\}$, $\{q\} = \{v\}$ we obtain a system of 12 first degree differential equations with 12 unknown terms.

We can test the solving of this first degree differential equations by Runge-Kutta method. The attempts will be in vain, as the solution quickly diverge after only few steps, due to the calculation of errors transmitted because of the algorithm , reason for which we chose to use the modal analysis method.

For results at least as good we propose the approximation of these equations by cubic spleen functions , using as unknown terms the inertia moments which interfere in the system .In this case the continuity conditions for the first differentials assure the oneness and the convergence of the solution, because the matrix of the linear system becomes diagonal dominant .

We start with the sums:

$$S_\Delta''(x) = M_{i-1} \cdot \frac{x_i - x}{h_i} + M_i \cdot \frac{x - x_{i-1}}{h_i}, \qquad (17)$$

$$h_i = x_i - x_{i-1},$$

where

$$\Delta : a = x_0 \le x_1 \le x_2 \le \dots \le x_n = b,$$

when by two times integrating, according to some initial conditions and taking into account some continuity conditions we obtain a linear system of a particular form.

In addition there are indicated some resistance studies. Such studies are recommended for different automobile sub-assemblies such as chassis, mudguard, center piece and piston rod .

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Recomandat spre publicare: 15.05.2007