DEFLECTION OF CYLINDRICAL COVERING OF BREAKER DRUMS

L.C. Hanganu, C. Vîlcu, Fl. Tudose-Sandu-Ville
Technical University „Gh.Asachi" of Iaşi

INTRODUCTION

A computation method for deflection in cylindrical coverings of breaker drums with circular rigidity ribs is discussed, the deflection being induced by pressure exerted on the outer packing.

Reference is made to the theory of beams on elastic ground supporting a distributed load (pressure) and concentrated loads (the reactions in the rigidity ribs) the results being achieved by Krillof's method.

CONTENTS

The next calculus assumptions are admitted:
- the covering is considered like a cylindrical tank with thin wall having a constant thickness *h*;
- the covering supports are considered perfect stiff and allow its movements on the axial direction;
- the radial thickness *h* and the widths of the stiff rings are considered with constant values for each ring;
- it is ignored the interval covering stress;
- the lining pressure *p* exercised on the breaker drum covering has the same value on the entire surface of the covering.

The calculations are made for a part of the covering with *h* thickness equivalent with the unit - figure 1. This part is considered like catched beam to its heads loaded with an uniform distributed stress and *Q* forces concentrated (the elastic reactions of the stiff rings). In this situation, the neutral fiber equation is:

$$D_c \frac{d^4 y}{dx^4} + Ky = p(x) \qquad (1)$$

in which D_c is the cylindrical rigidity at the covering flexion, and *K* is the elastic medium rigidity; there are defined such as:

$$D_c = \frac{E \cdot h^2}{12\left(1-\mu^2\right)} \qquad (2)$$

$$K = \frac{E \cdot h}{r^2} \qquad (3)$$

The symbol *p(x)* show that change depends by values of *x* - coordinate (interface here the pressure and the forces Q_i); *E* is the elasticity module and μ is Poisson coefficient.

It is devided the relation (1) with D_c; it is noted $K/D_c = 4 \cdot \beta^4$ and once again the same relation is devided with β^4. The equation resulted is:

$$\frac{d^4 y}{d\xi^4} + 4y = \frac{4p(\xi)}{K} \qquad (4)$$

with $\xi = \beta \cdot x$ a reduced *x* - coordinate.

The differential equation solution, using Krillof's method, is:

$$y = A \cdot Y_1(\xi) + B \cdot Y_2(\xi) + C \cdot Y_3(\xi) + D \cdot Y_4 \xi + \phi(\xi) \qquad (5)$$

in which the Krikllof's functions are $Y_i(\xi)$; *A*, *B*, *C*, *D* are constants integration; $\phi(\xi)$ is a function depending on the beam charge.

For a charge only with distributed load *p* or only with a concentrated force Q_i the functions ϕ_i, the functions $\phi(\xi)$ are:

$$\phi_p(\xi) = \frac{p}{K}\left[1 - Y_1(\xi)\right] \qquad (6)$$

$$\phi_{Qi}(\xi) = \frac{4\beta Q_i}{K} Y_4(\xi - \alpha_1) \qquad (7)$$

in which $\alpha_i = \beta \cdot x_i$.

For the beam with one fixed end ($\xi = 0$) and having the restriction:

$$\xi = 0 \begin{cases} y = 0 \\ y' = 0 \end{cases} \qquad (8)$$

and applying the properties of the function $Y_i(\xi)$, we have A=B=0.

Admiting, for exemple an add number n of circular ribs, at the middle of the beam will be the rib with m = (n+1)/2 index, such as it is shown in figure 1.

The Krillof's functions are:

$$\begin{aligned} Y_1(\xi) &= ch\xi \cdot cos\,\xi \\ Y_2(\xi) &= 0{,}5 \cdot (ch\xi \cdot sin\,\xi + sh\xi \cdot cos\,\xi) \\ Y_3(\xi) &= 0{,}5 \cdot sh\xi \cdot sin\,\xi \\ Y_4(\xi) &= 0{,}25 \cdot (ch\xi \cdot sin\,\xi - sh\xi \cdot cos\,\xi) \end{aligned} \qquad (9)$$

For x = 0 (ξ=0) the Krillof's functions properties are:

$$\begin{aligned} Y_1'(0) &= 1 \\ Y_2'(0) &= Y_3'(0) = Y_4'(0) = 0 \end{aligned} \qquad (10)$$

There are established the next connections between Krillof's functions:

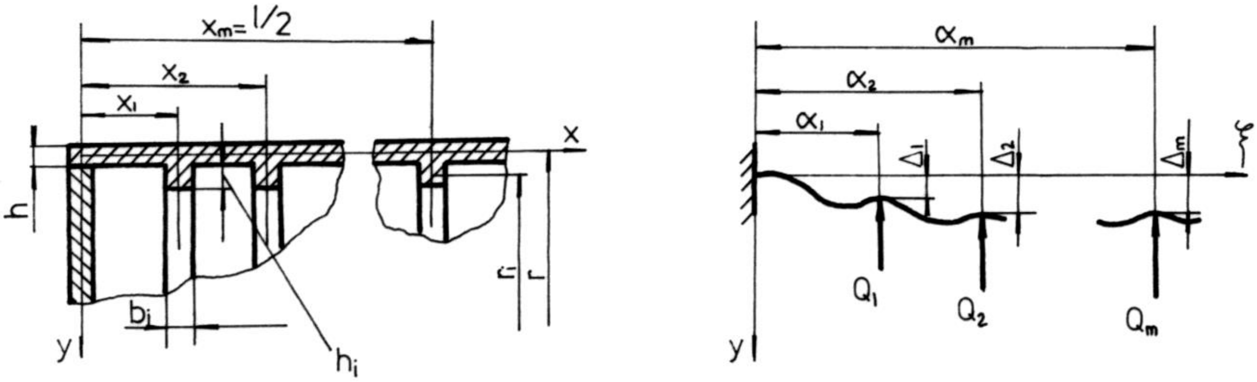

Figure 1. The deflection schema of cylindrical coverings of breaker drums.

$$Y_1'(\xi) = -4\beta Y_4(\xi);\; Y_2'(\xi) = \beta Y_1(\xi);\; Y_3'(\xi) = \beta Y_2(\xi);\; Y_4'(\xi) = \beta Y_3(\xi) \tag{11}$$

For the mentioned situation, the neutral fibre equations are:

$$y_1 = CY_3(\xi) + DY_4(\xi) + \frac{p}{K}[1 - Y_1(\xi)], \qquad \text{for } 0 \le \xi \le \alpha_1$$

$$y_2 = CY_3(\xi) + DY_4(\xi) + \frac{p}{K}[1 - Y_1(\xi)] + \frac{4\beta Q_1}{K} Y_4(\xi - \alpha_1), \qquad \text{for } \alpha_1 \le \xi \le \alpha_2$$

....

$$y_m = CY_3(\xi) + DY_4(\xi) + \frac{p}{K}[1 - Y_1(\xi)] - \frac{4\beta}{K}\cdot[(Q_1 Y_4(\xi - \alpha_1) + Q_2 Y_4(\xi - \alpha_2) + ... + Q_{m-1} Y_4(\xi - \alpha_{m-1}))] \qquad \text{for } \alpha_{m-1} \le \xi \le \alpha_m$$

(12)

There are here $m+2$ unknowns: $C, D, Q_1, Q_2, ..., Q_m$, it is necessary a system with $m+2$ conditions equations:

$$\begin{cases} \text{for} \quad \xi = \alpha_m \quad \begin{cases} y'(\alpha_m) = 0 \\ D_c y_m'''(\alpha_m) = -\dfrac{Q_m}{2} \end{cases} \\ \Delta_1 = \dfrac{Q_1}{K_{i1}} \\ .... \\ \Delta_m = \dfrac{Q_m}{K_{im}} \end{cases} \quad (m+2) \tag{13}$$

in which Δ_i represents the radial deformation of the rings if they are being stressed by the Q_i forces; K is the ring rigidity:

Using the conditions (13), we find a $m+2$ equations system:

$$K_i = \frac{A_i E}{r_i^2} \tag{14}$$

in which $A_i = b_i \cdot h_i$ is the surface of ring transversal section, r_i – the average radius, E – elasticity module for ring material.

$$CY_2(\alpha_m) + DY_3(\alpha_m) + \frac{4p}{K} Y_4(\alpha_m) - \frac{4\beta}{K}\cdot[Q_1 Y_3(\alpha_m - \alpha_1) + Q_2 Y_3(\alpha_m - \alpha_2) + ... + Q_{m-1} Y_3(\alpha_m - \alpha_{m-1})] = 0$$

$$-4CY_4(\alpha_m) + DY_1(\alpha_m) + \frac{4p}{K} Y_2(\alpha_m) - \frac{4\beta}{K}[Q_1 Y_1(\alpha_m - \alpha_1) + Q_2 Y_1(\alpha_m - \alpha_2)] +$$
$$+ \frac{4\beta}{K}\left[+ Q_{m-1} Y_1(\alpha_m - \alpha_{m-1}) + \frac{Q_m}{2}\right] = 0$$

$$CY_3(\alpha_1) + DY_4(\alpha_1) + \frac{p}{K}[1 - Y_1(\alpha_1)] - \frac{Q_1}{K_{il}} = 0$$

$$CY_3(\alpha_2) + DY_4(\alpha_2) + \frac{p}{K}[1 - Y_1(\alpha_2)] - \frac{4b}{K} Q_1 Y_4(\alpha_2 - \alpha_1) - \frac{Q_1}{K_{i2}} = 0$$

....

$$CY_3(\alpha_m)+DY_4(\alpha_m)+\frac{p}{K}[1-Y_1(\alpha_m)]-\cdot$$
$$-\frac{4\beta}{K}[Q_1Y_4(\alpha_m-\alpha_1)+Q_2Y_4(\alpha_m-\alpha_2)+...+Q_{m-1}Y_4(\alpha_m-\alpha_{m-1})]-\frac{Q_m}{K_{im}}=0 \qquad (15)$$

With the help of (15) system, containig $m+2$ liniar equations with free term, we obtain the values for $C, D, Q_1, ..., Q_m$, the other values ($K, \beta, K_i, \alpha_1,... \alpha_m$) are determined from initial data ($r, h, l, E, r_i, h_i, x_i, p$).

With the values established for $C, D, Q_1,..., Q_m$, using the relation (11), we calculate the radial deformation for cylindrical covering; there are adopted for ξ the values concerning the middle zones from two consecutive circular ribs.

It is necessary that the maximum value for radial deformation calculated to be less than adopted admissible value.

CALCULUS EXEMPLE

Now it is presented a calculus exemple for a breaker drum with steel covering. The initial data is: r =638,5 mm; h = 5 mm, E = $2,1\cdot10^5$ N/mm^2, 7 identical OL steel circular ribs with b_i = 30 mm, h_i = 22 mm and r_i = 625 mm, x_i = 115 mm, x_2 = 225 mm, x_4 = 445 mm, p = 0,087 N/mm^2 the extent force for the rigid lining is by 50N, the rigid lining base is by 0,9 mm.

The intermediary values, strictly necessary for the system solutions (15) are;
K = 2,56 N/mm^3; β = 0,0256 mm^{-1}, k_1 = 354 N/mm^2, α_1 = 2,9, α_2 = 5,8, α_3 = 8,7, α_4 = 11,6.

The main determintive for the system is:

$$\begin{vmatrix}
Y_2(\alpha_4) & Y_3(\alpha_4) & -\frac{4\beta}{K}Y_3(\alpha_4-\alpha_1) & -\frac{4\beta}{K}Y_3(\alpha_4-\alpha_2) & -\frac{4\beta}{K}Y_3(\alpha_4-\alpha_3) & 0 \\
-Y_4(\alpha_4) & Y_1(\alpha_4) & -\frac{4\beta}{K}Y_1(\alpha_4-\alpha_1) & -\frac{4\beta}{K}Y_1(\alpha_4-\alpha_2) & -\frac{4\beta}{K}Y_1(\alpha_4-\alpha_3) & -\frac{2\beta}{K} \\
Y_3(\alpha_1) & Y_4(\alpha_1) & -\frac{1}{K_{11}} & 0 & 0 & 0 \\
Y_3(\alpha_2) & Y_4(\alpha_2) & -\frac{4\beta}{K}Y_4(\alpha_2-\alpha_1) & -\frac{1}{K_{12}} & 0 & 0 \\
Y_3(\alpha_3) & Y_4(\alpha_3) & -\frac{4\beta}{K}Y_4(\alpha_3-\alpha_1) & -\frac{4\beta}{K}Y_4(\alpha_4-\alpha_2) & -\frac{1}{K_{13}} & 0 \\
Y_3(\alpha_3) & Y_4(\alpha_4) & -\frac{4\beta}{K}Y_4(\alpha_4-\alpha_1) & -\frac{4\beta}{K}Y_4(\alpha_4-\alpha_2) & -\frac{4\beta}{K}Y_4(\alpha_4-\alpha_3) & -\frac{1}{K_{14}}
\end{vmatrix}$$

The columns are coresponding for C, D, Q_1, Q_2, Q_3 and Q_4. The free terms column is:

$$\begin{cases}
-\frac{4p}{K}Y_4(\alpha_4), \\
-\frac{4p}{K}Y_2(\alpha_4), \\
-\frac{p}{K}[1-Y_1(\alpha_1)], \\
-\frac{p}{K}[1-Y_1(\alpha_2)], \\
-\frac{p}{K}[1-Y_1(\alpha_3)], \\
-\frac{p}{K}[1-Y_1(\alpha_4)],
\end{cases}$$

Using Gauss method and a special computer programe, with the mentioned values, we obtain:

C = 0,0065775, D = -0,0143243, Q_1 = 46,39 N, Q_2 = 45,72 N, Q_3 = 45,4 N, Q_4 = 44,68 N.

With these values, using the relations (12), there are obtained the radial deformations in the middle of the distance between two consecutive circular ribs: $\Delta_{0,1}$ = 0,02 mm; $\Delta_{1,2}$ = 0,023 mm; $\Delta_{2,3}$ = 0,023 mm, $\Delta_{3,4}$ = 0,023 mm.

References

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