TOWARDS A DIAGNOSTICATION METHOD OF ANXIETY USING THE CHOQUET INTEGRAL

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Abstract: *This article describes a mathematical model through which the level of EEG type waves are processed in order to characterize the level of anxiety. Our idea is to use the Choquet integral with respect to a monotone measure. We consider the data resulting from the EEG wave measurements for a group of subjects. We describe a procedure by using different monotone measures to calculate the anxiety level of a subject using the Choquet integral. For each patient we have the level of anxiety given by psychologists. For each patient we compare the results obtained by this method with the results of psychologists. Of all the measures used, we chose the measure that provided the closest results to the real ones.*

Key Words: *Anxiety, EEG, Choquet integral, monotone measure, Big Five, C++.*

INTRODUCTION

In this paper we describe a mathematical model through which the level of EEG type waves is processed in order to characterize the level of anxiety, which represents changes in the values of the personality characteristics of the BigFive model. The goal was to determine a mathematical tool through which to diagnose the level of anxiety.

In writing this article we have worked in collaboration with the Institute of Studies, Research, Development and Innovation of Titu Maiorescu Faculty in Bucharest, as well as with specialists of the Military Technical Academy in Bucharest.

We considered the data resulting from the EEG wave measurements. The measurements of the values of EEG waves were measured in 14 subjects. In order to carry out the measurements a NeuroSky device, with two sensors, of which one active was used. The specialists in psychology state that the anxiety is characterized by LowAlpha, HightAlpha, LowBeta and HightBeta waves.The input data used were specific values of EEG waves, as well as classification data of the anxiety level, provided by the Psychology Research Institute. The classification is given by numbers from 0 to 100.

As a mathematical procedure, the nonlinear integrals are used as a fusion instrument. We used 8 monotone measures to calculate the anxiety level of each subject, using the Choquet integral. For each measure, we compared the results obtained by our method with the results of psychologists. Of all the measures used, we chose the measure that provided the closest results to the real ones.

To calculate the values, we have created a C++ programme. The source code is written in C++ in the CodeBlocks development medium, 17.12 version on Windows 10 operating system, combined with GNU GCC Compiler in MinGW distribution, 6.3 version. For the matrix operations the Eigen library, version 3.3 was used.

Finally, we have determined a monotonous measure to provide the closest results in relation to psychological results in terms of anxiety.

The determined instrument will be used to draw conclusions regarding the level of anxiety of other subjects who have been measured with NeuroSky.

In the sequel we explain how the nonlinear integral was used for the aggregation of data for the above-mentioned problem, and, of course, we explain the obtained results.

1. PRELIMINARY FACTS

Definition 2.1: A measurable space is a couple (T, τ), where T is a non-empty set and $\tau \subset \mathcal{P}(T)$ is a σ-algebra.

Definition 2.2: If (T, τ) is a measurable space, a monotone measure is a function $\mu: \tau \rightarrow \mathbb{R}_+$ having the properties: i) $\mu(\emptyset) = 0$; ii) $\mu(A) \leq \mu(B)$ for any A, B in τ such that $A \subset B$.

Definition 2.3: The Choquet integral of the function f with respect to the measure μ is the element $\int_0^\infty \mu(F_a)da \in \overline{\overline{\mathbb{R}_+}}$. We shall write: $(C)\int f d\mu = \int_0^\infty \mu(F_a)\, da$.

We shall say that f is Choquet integrable with respect to μ in case $(C)\int f d\mu < \infty$.

F_a={t ∈ T |f(t)≥ a} = $f^{-1}([a,\infty))\in\tau$.

Special formula 2.4: If T is finite, $T=\{x_1,x_2,\dots,x_n\}$, *n≥1*,
(C)$\int f d\mu=\sum_{i=1}^{n}(f(x_i^*)-f(x_{i-1}^*))\mu(\{x_i^*,x_{i+1}^*,\dots,x_n^*\})$ with the convention $f(x_0^*)=0$.

2. DETERMINATION OF THE ANXIETY DEGREE

We made l=14 measurements. These are the l=14 functions $f_1,f_2,\dots f_{14}$. The n=4 measured attributes are $x_1=LowAlpha, x_2=HighAlpha, x_3=LowBeta, x_4=HighBeta.$ Namely, for each of the 14 measurements (rows), we obtained the input values $f_p(x_1),f_p(x_2),f_p(x_3),f_p(x_4)$ and the output values $y_p, p=1,2,\dots,14$. So, the fifth column contains the output values $y_p, p=1,2,\dots,14$. The input values are the averages of the measurements carried on the 14 subjects. The output values are obtained using the classification given by the psychologists to the subjects (measured individuals). These output values are represented by grades, from 0 to 100.

In the C++ program, we used a function to process the data from the CSV files, and to create a matrix.

Thus, we obtained the table (T_1), with 14 rows and 4+1=5 columns.

In order to save typographical space, we exhibit below only one row of the table (T_1):

Number of Subject	Low Alpha	High Alpha	Low Beta	High Beta	Grade
S1	33738.85	26911.79	15911.23	15827.22	10

Psychological results:
S1=10; S2=0; S3=5; S4=9; S5=11; S6=5; S7=10; S8=62; S9=10; S10=29; S11=10; S12=12; S13=13; S14=11.

We considered $t=8$ monotone measures. Using each measure μ_k ($k=\overline{1,t}$), for each subject p, we calculated the level of anxiety $z_{k,p}$ ($k=1,\dots,t$ and $p=1,\dots,14$).As we have said, we decided to choose as fusion instrument the Choquet integral of the functions $f_p, p=1,2,\dots,14$, with respect to a monotone measure μ_k. So, for any $k=\overline{1,t}$ and for any $p=\overline{1,14}$ one has:

$$z_{k,p}=(C)\int f_p d\mu_k$$

For each measure μ_k, we compared the results obtained by our method with the results of psychologists, using The Least Squares Method. Actually, for each measure μ_k, we calculated $E_k=\sum_{p=1}^{14}(z_{k,p}-y_p)^2$.

We considered the measures:

$$\mu_1(E)=\begin{cases}0, if\ E=\emptyset\\ 0.0000143333, if\ E=\{x_1\}\\ 0.0000406178, if\ E=\{x_2\}\\ 0.0000471675, if\ E=\{x_1,x_2\}\\ 0.00000472641,\ if\ E=\{x_3\}\\ 0.0000143333, if\ E=\{x_1,x_3\}\\ 0.0000406178, if\ E=\{x_2,x_3\}\\ 0.0000955576, if\ E=\{x_1,x_2,x_3\}\\ 0.0000254858, if\ E=\{x_4\}\\ 0.0000320333, if\ E=\{x_1,x_4\}\\ 0.0000406178, if\ E=\{x_2,x_4\}\\ 0.0000471675, if\ E=\{x_1,x_2,x_4\}\\ 0.000225524, if\ E=\{x_3,x_4\}\\ 0.000225524, if\ E=\{x_1,x_3,x_4\}\\ 0.000616894, if\ E=\{x_2,x_3,x_4\}\\ 0.000616894, if\ E=\{x_1,x_2,x_3,x_4\}\end{cases}, \mu_2(E)=\begin{cases}0, if\ E=\emptyset\\ 0.0000245333, if\ E=\{x_1\}\\ 0.0000507178, if\ E=\{x_2\}\\ 0.00671685, if\ E=\{x_1,x_2\}\\ 0.00000972621,\ if\ E=\{x_3\}\\ 0.000246333, if\ E=\{x_1,x_3\}\\ 0.000606178, if\ E=\{x_2,x_3\}\\ 0.01955576, if\ E=\{x_1,x_2,x_3\}\\ 0.0000324858, if\ E=\{x_4\}\\ 0.000329333, if\ E=\{x_1,x_4\}\\ 0.00606178, if\ E=\{x_2,x_4\}\\ 0.051675, if\ E=\{x_1,x_2,x_4\}\\ 0.0625524, if\ E=\{x_3,x_4\}\\ 0.725524, if\ E=\{x_1,x_3,x_4\}\\ 0.816894, if\ E=\{x_2,x_3,x_4\}\\ 0.916894, if\ E=\{x_1,x_2,x_3,x_4\}\end{cases},$$

$$\mu_3(E)=\begin{cases}0, if\ E=\emptyset\\ 0.000014, if\ E=\{x_1\}\\ 0.00004, if\ E=\{x_2\}\\ 0.000047, if\ E=\{x_1,x_2\}\\ 0.0000047,\ if\ E=\{x_3\}\\ 0.0000145, if\ E=\{x_1,x_3\}\\ 0.000048, if\ E=\{x_2,x_3\}\\ 0.000095, if\ E=\{x_1,x_2,x_3\}\\ 0.000025, if\ E=\{x_4\}\\ 0.000032, if\ E=\{x_1,x_4\}\\ 0.0000406, if\ E=\{x_2,x_4\}\\ 0.0000471, if\ E=\{x_1,x_2,x_4\}\\ 0.0002, if\ E=\{x_3,x_4\}\\ 0.00023, if\ E=\{x_1,x_3,x_4\}\\ 0.000616, if\ E=\{x_2,x_3,x_4\}\\ 0.0006168, if\ E=\{x_1,x_2,x_3,x_4\}\end{cases},\ \mu_4(E)=\begin{cases}0, if\ E=\emptyset\\ 0.001, if\ E=\{x_1\}\\ 0.00004, if\ E=\{x_2\}\\ 0.002, if\ E=\{x_1,x_2\}\\ 0.0003,\ if\ E=\{x_3\}\\ 0.003, if\ E=\{x_1,x_3\}\\ 0.0005, if\ E=\{x_2,x_3\}\\ 0.006, if\ E=\{x_1,x_2,x_3\}\\ 0.00002, if\ E=\{x_4\}\\ 0.004, if\ E=\{x_1,x_4\}\\ 0.006, if\ E=\{x_2,x_4\}\\ 0.01, if\ E=\{x_1,x_2,x_4\}\\ 0.02, if\ E=\{x_3,x_4\}\\ 0.03, if\ E=\{x_1,x_3,x_4\}\\ 0.04, if\ E=\{x_2,x_3,x_4\}\\ 0.05, if\ E=\{x_1,x_2,x_3,x_4\}\end{cases},$$

$$\mu_5(E)=\begin{cases}0, if\ E=\emptyset\\ 0.03, if\ E=\{x_1\}\\ 0.1, if\ E=\{x_2\}\\ 0.2, if\ E=\{x_1,x_2\}\\ 0.02,\ if\ E=\{x_3\}\\ 0.04, if\ E=\{x_1,x_3\}\\ 0.3, if\ E=\{x_2,x_3\}\\ 0.4, if\ E=\{x_1,x_2,x_3\}\\ 0.008, if\ E=\{x_4\}\\ 0.07, if\ E=\{x_1,x_4\}\\ 0.5, if\ E=\{x_2,x_4\}\\ 0.6, if\ E=\{x_1,x_2,x_4\}\\ 0.7, if\ E=\{x_3,x_4\}\\ 0.8, if\ E=\{x_1,x_3,x_4\}\\ 0.9, if\ E=\{x_2,x_3,x_4\}\\ 0.95, if\ E=\{x_1,x_2,x_3,x_4\}\end{cases},\ \mu_6(E)=\begin{cases}0, if\ E=\emptyset\\ 0.21, if\ E=\{x_1\}\\ 0.22, if\ E=\{x_2\}\\ 0.3, if\ E=\{x_1,x_2\}\\ 0.009,\ if\ E=\{x_3\}\\ 0.35, if\ E=\{x_1,x_3\}\\ 0.429, if\ E=\{x_2,x_3\}\\ 0.54, if\ E=\{x_1,x_2,x_3\}\\ 0.01, if\ E=\{x_4\}\\ 0.25, if\ E=\{x_1,x_4\}\\ 0.26, if\ E=\{x_2,x_4\}\\ 0.47, if\ E=\{x_1,x_2,x_4\}\\ 0.1, if\ E=\{x_3,x_4\}\\ 0.6, if\ E=\{x_1,x_3,x_4\}\\ 0.7, if\ E=\{x_2,x_3,x_4\}\\ 0.8, if\ E=\{x_1,x_2,x_3,x_4\}\end{cases},$$

$$\mu_7(E)=\begin{cases}0, if\ E=\emptyset\\ 0.91, if\ E=\{x_1\}\\ 0.8, if\ E=\{x_2\}\\ 0.92, if\ E=\{x_1,x_2\}\\ 0.94,\ if\ E=\{x_3\}\\ 0.95, if\ E=\{x_1,x_3\}\\ 0.96, if\ E=\{x_2,x_3\}\\ 0.97, if\ E=\{x_1,x_2,x_3\}\\ 0.98, if\ E=\{x_4\}\\ 0.99, if\ E=\{x_1,x_4\}\\ 0.995, if\ E=\{x_2,x_4\}\\ 0.997, if\ E=\{x_1,x_2,x_4\}\\ 0.998, if\ E=\{x_3,x_4\}\\ 0.9984, if\ E=\{x_1,x_3,x_4\}\\ 0.9989, if\ E=\{x_2,x_3,x_4\}\\ 0.99993, if\ E=\{x_1,x_2,x_3,x_4\}\end{cases},\ \mu_8(E)=\begin{cases}0, if\ E=\emptyset\\ 0.0123, if\ E=\{x_1\}\\ 0.6178, if\ E=\{x_2\}\\ 0.7675, if\ E=\{x_1,x_2\}\\ 0.0004,\ if\ E=\{x_3\}\\ 0.02, if\ E=\{x_1,x_3\}\\ 0.62, if\ E=\{x_2,x_3\}\\ 0.79, if\ E=\{x_1,x_2,x_3\}\\ 0.00002, if\ E=\{x_4\}\\ 0.1, if\ E=\{x_1,x_4\}\\ 0.65, if\ E=\{x_2,x_4\}\\ 0.83, if\ E=\{x_1,x_2,x_4\}\\ 0.85, if\ E=\{x_3,x_4\}\\ 0.879, if\ E=\{x_1,x_3,x_4\}\\ 0.895, if\ E=\{x_2,x_3,x_4\}\\ 0.9, if\ E=\{x_1,x_2,x_3,x_4\}\end{cases}.$$

For *k=1*, we obtained the conclusions: $z_{1,1}=10.3885$, $z_{1,2}$ =0.000856597, $z_{1,3}$ =5.40384, $z_{1,4}=8.57645, z_{1,5}=11.089, z_{1,6}=4.8107, z_{1,7}=9.96549, z_{1,8}=62.273, z_{1,9}=9.59623, z_{1,10}=29.2099, z_{1,11}=10.3885, z_{1,12}=11.8409, z_{1,13}=12.7664, z_{1,14}=11.1403$.

And $E_1=\sum_{p=1}^{14}(z_{1,p}-y_p)^2$ = 1.07048.

For *k=2*, we obtained the conclusions: $z_{2,1}$ =14587.6, $z_{2,2}$ =0.979476, $z_{2,3}=8515.13$, $z_{2,4}=11491.3$, $z_{2,5}=15824.1$, $z_{2,6}=7071.18$, $z_{2,7}=13682$, $z_{2,8}=87487.3$, $z_{2,9}=13621.4$, $z_{2,10}=22975.4$, $z_{2,11}$ =14587.6

, $z_{2,12}=16786.8, z_{2,13}=17991.9, z_{2,14}=10585.7$. And $E_2=\sum_{p=1}^{14}(z_{2,p}-y_p)^2$ =1.01876e+010

For *k=3*, we obtained the conclusions: $z_{3,1}=10.3828$, $z_{3,2}$ =0.0008309, $z_{3,3}=5.4095, z_{3,4}$ =8.56771, $z_{3,5}=$ 11.0831 $z_{3,6}$ =4.80914, $z_{3,7}=9.95662$, $z_{3,8}=62.2256$, $z_{3,9}=9.58957$, $z_{3,10}=29.0884$, $z_{3,11}=10.3828$, $z_{3,12}=11.8331, z_{3,13}=12.7554, z_{3,14}$ =11.1339.. And $E_3=\sum_{p=1}^{14}(z_{3,p}-y_p)^2$ =1.02563.

For *k=4*, we obtained the conclusions: $z_{4,1}$ =820.693, $z_{4,2}=0.0709$, $z_{4,3}$ =457.138, $z_{4,4}$ =672.667
, $z_{4,5}$ =911.819, $z_{4,6}$ =389.928, $z_{4,7}$ =791.865, $z_{4,8}=4777.8$, $z_{4,9}$ =798.017, $z_{4,10}$ =2275.92
, $z_{4,11}$ =820.693, $z_{4,12}$ =952.129, $z_{4,13}$ =1020.56, $z_{4,14}$ =1203.77.

And $E_4 = \sum_{p=1}^{14}(z_{4,p} - y_p)^2$ =3.47604e+007.

For $k=5$, we obtained the conclusions: $z_{5,1}$ =17474.4, $z_{5,2} = 1.71$, $z_{5,3}$ =9020.28, $z_{5,4}$ =15371.5, $z_{5,5}$ =20130.3, $z_{5,6}$ =7474.91, $z_{5,7}$ =17292.5, $z_{5,8} = 103691$, $z_{5,9} = 18037$, $z_{5,10} =$ 130079, $z_{5,11}$ =17474.4, $z_{5,12}$ =20050.9, $z_{5,13}$ =21066.4, $z_{5,14}$ =31171.6. And $E_5 = \sum_{p=1}^{14}(z_{5,p} - y_p)^2$ =3.14793e+010.

For $k=6$, we obtained the conclusions: $z_{6,1} = 17441$, $z_{6,2} = 0.927$, $z_{3,3}$ =7824.45, $z_{6,4}$ =16887.4, $z_{6,5}$ =19262.9, $z_{6,6}$ =6891.82, $z_{6,7}$ =18056.5, $z_{6,8}$ =88220.1, $z_{6,9}$ =17939.7, $z_{6,10} = 108013$, $z_{6,11} = 17441$, $z_{6,12}$ =20480.4, $z_{6,13}$ =23059.8, $z_{6,14}$ =47984.4. And $E_6 = \sum_{p=1}^{14}(z_{6,p} - y_p)^2$ =2.47029e+010.

For $k=7$, we obtained the conclusions: $z_{7,1}$ =32240.7 , $z_{7,2}$ =4.81793, $z_{7,3}$ =11709.2, $z_{7,4}$ =33475.2 , $z_{7,5}$ =34915.5, $z_{7,6}$ =10417.2, $z_{7,7}$ =32146.6, $z_{7,8} =$ 142794, $z_{7,9}$ =35057.9, $z_{7,10} = 348265$, $z_{7,11}$ =32240.7 , $z_{7,12}$ =39580.7, $z_{7,13}$ =48367.8, $z_{7,14} = 148793$. And $E_7 = \sum_{p=1}^{14}(z_{7,p} - y_p)^2$ = 1.74603e+011.

For $k=8$, we obtained the conclusions: $z_{8,1}$ =22837.8, $z_{8,2} = 1.7512$, $z_{8,3}$ =8782.34 , $z_{8,4}$ =20239.1, $z_{8,5}$ =23584.2, $z_{8,6}$ =7102.06, $z_{8,7}$ =19743.5, $z_{8,8} = 120627$, $z_{8,9}$ =20736.4, $z_{8,10} =$ 253117 , $z_{8,11}$ =22837.8, $z_{8,12} = 23527$, $z_{8,13} = 24008$, $z_{8,14}$ =39238.1. And $E_8 = \sum_{p=1}^{14}(z_{8,p} - y_p)^2$ =8.42105e+010.

We chose the measure that provided the closest results to the real ones. Actually, we chose the minimum value of E_k. That is E_3.

So, μ_3 is a monotonous measure which provides the closest results in relation to psychological results in terms of anxiety.

CONCLUSIONS

- The studied level of anxiety represents changes in the values of the personality characteristics in the BigFive model, and its values were determined using EEG waves.
- The determined instrument will be used to draw conclusions regarding the level of anxiety of other subjects who have been measured with NeuroSky.

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