Analysis of Non-Classical Heat Conduction Models

Irina Drin [1], Svitlana Drin [2], Yaroslav Drin [3], Mykhailo Lutskiv [4]

[1] Chernivtsi Institute of Trade and Economics of Kyiv National University of Trade and economics, ORCID: 0000-0002-0258-7007, irina_drin@ukr.net

[2] National University of "Kyiv-Mohyla Academy", ORCID: 0000-0002-5576-3756, svitlana.drin@gmail.com

[3] Yuriy Fedkovych Chernivtsi National University, ORCID: 0000-0003-0945-7325, y.drin@chnu.edu.ua

[4] Ternopil Ivan Puluj National Technical University, ORCID: 0009-0009-0361-1417

Keywords: heat conduction process, classical and generalized boundary value problems, computer modeling, numerical method

Abstract. Let $t > 0$ be the time variable, x – the spatial variable where $0 \leq x \leq l$ and f, φ, μ_1, μ_2 be known function of their arguments. Let $u(x,t)$ be unknown function representing temperature of a rod positioned along the *Ox* axis, where $0 \leq x \leq l$. Using computer modeling, the following heat conduction problems have been investigated:

- the first boundary value problem:

$$\frac{\partial u(x,t)}{\partial t} = a^2 \frac{\partial^2 u(x,t)}{\partial x^2} + f(x,t), \tag{1}$$

$$u(x,0) = \varphi(x), \tag{2}$$

$$u(0,t) = \mu_1(t), \tag{3}$$

$$u(l,t) = \mu_2(t), \tag{4}$$

- the generalized first boundary value problem: (1)-(3),

$$\frac{\partial u(l,t)}{\partial x} = \mu_2(t). \tag{4'}$$

Analytical and numerical solutions for these problems have been obtained. Graphs have been constructed for them (Fig. 1 and Fig. 2 respectively) with the known $\varphi(x), \mu_1, \mu_2$ in [1], [2], [3]

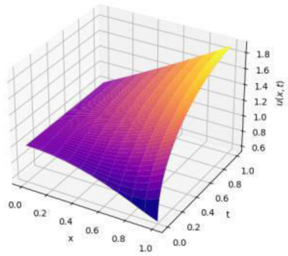

Fig. 1

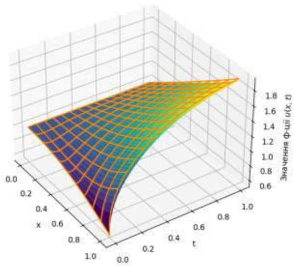

Fig. 2

The graphs of the numerical and analytical solutions coincide over the entire range of investigated time and space values.

Further improvements in the accuracy of the numerical solution can be achieved by adjusting grid parameters – reducing spatial step size and increasing the number of computational iterations.

References

[1] I. Drin, S. Drin, Y. Drin, M. Lutskiv, Non-classical boundary Value Problem For The Heat Conduction Equation, Problems of decision making under uncertainties: Brno, Czech Republic, 2024, p. 52

[2] I. Drin, S. Drin, Y. Drin, M. Lutskiv, Study of numerical and analytical solutions of a generalized boundary value problem for the heat conduction equation, Problems of decision making under uncertainties: Brno, Czech Republic, 2024, pp. 53-54

[3] N. Uhryniak, Mathematical modeling of heat conduction, Yuriy Fedkovych Chernivtsi National University, Bachelor thesis, 2024 (in Ukrainian)