Influence of Transversal Constant Electric Field on Mobility in Parabolic Quantum Well

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***Abstract* — The mobility μ in the electric field E directed perpendicularly to surface of the low-dimensional system with consideration for the interaction of the carriers with the rough surface and long-wave acoustic vibrations has been calculated. It is shown that μ reduces with E increasing and the influence of the processes of electron scattering on the rough surface can be substantial within the region of high temperatures as well.**

***Index Terms* — the parabolic quantum well, the transversal electric field, Kubo's formula, the scattering of carriers, the acoustic phonons, the rough surface.**

I. INTRODUCTION

In parabolic quantum wells (PQW) when the constant electric field E is directed along the spatial quantization axis the electron potential energy has a form:

$$U(z)=\frac{m\omega^2}{2}z^2+eEz\,.$$

Thus, with increasing of electric field intensity, potential *U(z)* is shifted into the region of negative values of *z* and drops by the value $\Delta_c=e^2E^2/2m\omega^2$. The wave function of Schrödinger's equation with potential energy *U(z)* is known [1], the eigenvalues of electron energy in the conduction band are determined by the relations:

$$E_{n,k_\perp}=\frac{\hbar^2k_\perp^{\ 2}}{2m}z^2+E_n\,, \qquad (1)$$

$$E_n=\hbar\omega\left(n+\frac{1}{2}\right)-\Delta_c\,,\ k_\perp^2=k_x^2+k_y^2\,,$$

where $\hbar\omega$ is the step of the energy quantization, which simply connected with the value of potential energy ΔE_c on the boundary of PQW with the width *a*, $\hbar\omega=\frac{2\hbar}{a}\sqrt{\frac{2\Delta E_c}{m}}$; $k_\perp$ - the electron wave vector in the plane of the low-dimension system.

As it immediately follows from (1) with increasing of **E** the bottom of conduction band falls into the region of the forbidden energy values by the magnitude Δ_c.

Hereafter we examine such values of the electric field intensity when the parabolic potential well keeps its form and there are still a lot of size-quantized equidistant levels. It is possible if the energy gained by the electron in the electric field when passing through the thickness of PQW is about or less than ΔE_c. For typical values of PQW parameters: $\Delta E_c\sim 0.25\,\text{eV}$ and $a=10^3$ Å, $E\le 2.5\cdot10^4$ V/cm.

In the low temperature region in the undoped low-dimension systems it is important the scattering of carriers on the rough surface [2-3].

Owing to the surface roughness of the quantum system the PQW width *a* is changed stochastically, i.e. the dimensionally quantization energy E_n fluctuates at the motion of electrons along the surface of quantum system. Thus, the energy of the interaction of the electron with the rough surface is written in the form [2]:

$$W=\frac{\partial E_n}{\partial L}\Delta(x,y)\equiv V_n\Delta(x,y)\,, \qquad (2)$$

where $\Delta(x,y)$ is the random function.

At the gauss fluctuations of surface the autocorrelation function for the different surface points is determined by the relation:

$$\{\Delta(x,y)\Delta(x',y')\}= \qquad (3)$$
$$=\Delta^2\exp\left[-\frac{\left((x-x')^2+(y-y')^2\right)}{\Lambda^2}\right]\equiv F^{(G)}\left(|\boldsymbol{\rho}-\boldsymbol{\rho}'|\right)$$

where $|\boldsymbol{\rho}-\boldsymbol{\rho}'|=\left[(x-x')^2+(y-y')^2\right]^{\frac{1}{2}}$; Δ is the height of the gauss fluctuation; Λ is its length; $\{\ \}$ means the averaging on the realizations of the random process $\Delta(x,y)$.

For the δ-shaped fluctuation of surface:

$$\{\Delta(x,y)\Delta(x',y')\}= \qquad (4)$$
$$=\gamma\delta(x-x')\delta(y-y')\equiv F^{(\delta)}\left(|\boldsymbol{\rho}-\boldsymbol{\rho}'|\right)$$

Note that at the low temperatures the type of the surface fluctuations weakly influences on the final results of the calculated observed physical quantity (for instance, the electric conductivity).

II. BASIC CONSIDERATION

The calculation of the electric conductivity is carried out using Kubo's formula [4]. In the approximation of the relaxation time [5] the final expression for electric

conductivity is written in the form:

$$\sigma_{xx} = \frac{e^2}{k_0 T V m^2} \sum_{\alpha,\alpha_1} P^{(x)}_{\alpha\alpha_1} P^{(x)}_{\alpha\alpha_1} \tau_\alpha n_\alpha \left(1 - n_{\alpha_1}\right), \quad (5)$$

where α and α_1 are the quantum numbers describing the electron states; V is the volume of the basic region of the dimensionally quantized system; $P^{(x)}_{\alpha\alpha_1}$ is the matrix element of the x^{th} component of the momentum operator on the wave functions of the electron in the conduction band; T is the temperature; n_α is the equilibrium function of distribution of carriers with energy $E_{n,k_\perp}$; $1/\tau_\alpha$ is determined by quantum-mechanical probability of the surface roughness scattering of carriers.

Thus:

$$\frac{1}{\tau_\alpha} = \frac{2\pi}{\hbar} \sum_\beta \widetilde{W}_{\alpha\beta} \delta\left(E_\alpha - E_\beta\right), \quad (6)$$

$$\widetilde{W}_{\alpha\beta} = \int d\mathbf{r}_1 d\mathbf{r}_2 \Psi^*_\alpha(\mathbf{r}_1) \Psi^*_\beta(\mathbf{r}_2) F(\mathbf{r}_1, \mathbf{r}_2) \Psi_\alpha(\mathbf{r}_2) \Psi_\beta(\mathbf{r}_1)$$

where $\Psi_i(\mathbf{r})$ ($i = \alpha, \beta$) are the wave functions of the electron in PQW in the transversal electric field.

For the case of the δ-shaped fluctuation of surface it is not difficult to obtain:

$$\frac{1}{\tau_\alpha} = \frac{8\gamma\Delta E_c}{\hbar a^4} \left[\left(n + \frac{1}{2}\right) + N_c\right]^2, \; N_c = \frac{2\Delta_c}{\hbar\omega}. \quad (7)$$

The calculation for the case of the gauss fluctuation of surface at low temperatures leads to the similar result as in (7) where it is necessary to replace γ by ($\pi\Delta^2\Lambda^2$). Note that the relaxation time (7) accurately coincides in the case under consideration with the relaxation transport time introduced in the integral of collisions when solving Boltsman's equation. As follows from (7) the relaxation time depends only on the number of the dimensionally quantized conduction band ($\tau_\alpha = \tau_n$). After summing over the electron wave vectors the electric conductivity is written as:

$$\sigma_{xx} = \frac{e^2}{a\pi\hbar^2\beta} \sum_n \tau_n \ln\left(1 + e^{-\beta\xi_n}\right), \quad (8)$$

$$\xi_n = E_n - \xi, \; \beta = \frac{1}{k_0 T},$$

where ξ is the chemical potential.

For nondegenerate electron gas ($\beta\xi_n >> 1$) at low temperatures when all carriers are in the lowest dimensionally quantized band ($n = 0$) the mobility is determined by the relation:

$$\mu_{xx} = \mu_{xx}(0) \frac{1}{\left(1 + 2N_c\right)^2}, \quad (9)$$

$$\mu_{xx}(0) = \frac{e}{m} \left(\frac{\hbar a^4}{2\gamma\Delta E_c}\right),$$

where $\mu_{xx}(0)$ is the mobility in PQW without the transversal electric field.

III. DISCUSSION AND CONCLUSION

For the typical parameters of PQW ($\Delta E_c = 0.25\,\text{eV}$, $m = 0.06 m_0$) $\hbar\omega = \frac{14.5}{a_0}\,\text{eV}$ (a_0 is the width of PQW in angstroms), $N_c = 1.7 \cdot 10^{-18} E_0^2 a_0^3$ (E_0 is measured in V/cm). Thus, for PQW with $a = 10^3$ Å, $E_0 = 2.5 \cdot 10^4$ V/cm, $N_c = 1$, the mobility decreases by nearly an order of magnitude.

With rising of temperature the scattering of carriers on long-wave acoustic oscillations begins to influence the mobility value. At elastic scattering of electrons, being in the lowest dimensionally quantized band $n = 0$ ($\hbar\omega >> k_0 T$), on the phonons within the high temperature region ($N_q \approx \frac{k_0 T}{h\nu q} >> 1$) the relaxation time has the form:

$$\frac{1}{\tau_f} = \left(\frac{m\omega}{2\pi\hbar}\right)^{\frac{1}{2}} \frac{E_1^2 m k_0 T}{\hbar^3 v^2 \rho}, \quad (10)$$

where E_1 is the constant of the deformation potential, ρ is the density of the examined quantum system, v is the speed of sound.

Note that τ_f doesn't depend on the transversal electric field intensity and electron wave vectors. Thus, the electric conductivity with the consideration for the scattering of carriers on the rough surface (τ_0) and the acoustic phonons (τ_f) is determined by the relation (8), in which $\frac{1}{\tau_n} = \frac{1}{\tau_0} + \frac{1}{\tau_f}$. The final expression for mobility in the case under examination is written in the following form:

$$\mu_{xx} = \mu_{xx}(0) \frac{1}{\left(1 + 2N_c\right)^2 + \Delta}, \quad (11)$$

$$\Delta = \left(\frac{m\omega}{2\pi\hbar}\right)^{\frac{1}{2}} \left(\frac{E_1}{\hbar\omega}\right)^2 \frac{4 k_0 T a^2}{\rho v^2 \gamma}.$$

As it immediately follows from (11) in the presence of the transversal constant electric field the influence of the processes of the surface roughness scattering of electrons remains substantially in the region of the higher temperatures.

At increasing of the transversal electric field intensity the bottom of the conduction band goes down in to the region of the forbidden energy values by the quantity Δ_c, and the valence band extremum rises $\Delta_v = e^2 E^2 / 2 m_v \omega_v^2$ ($\hbar\omega_v$ is the step of the dimensional quantization in the valence

band). Thus, the forbidden band width E_g of low-dimensional system is reduced by $\Delta_c + \Delta_v$. This very fact is the reason that when **E** increasing the one-band approximation could be broken and in order to describe kinetic phenomena in narrow-band semiconductor systems it is necessary to take into consideration the interaction between the conduction band and the valence bands. In the simplest Kane's model due to the conduction band being nonstandard the relaxation time τ_f becomes noticeably dependent on **E**. Namely for this reason the mobility starts to depend on the transversal electric field intensity in a complicated way.

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