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Reliability of Information-Computer Systems with Hierarchical Structure

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Abstract

A mathematical model of information-computer systems with hierarchical structure is built in general assumptions concerning the distributions of failure and repair times of the systems unit.

Keywords: systems with hierarchical structure, reliability, semi-Markov processes.

1 Introduction

The Information-Computer Systems have in main a specific structure: the information from some principal control unit is forthcoming to several next units, each of that in its turn communicates the information to the following units, etc. This structure takes place in systems in which the circular transmission of information signals (or management signals) or collection of information from downstairs elements are carried out.

2 Systems with Hierarchical Structure

We will consider the system S with hierarchical structure: the principal unit a_0 is connected with a_1 units of the first level, each of that is connected with a_2 units of the second level, etc. The units of the last level n-level are called extreme elements, their number is $N = a_1 \cdot a_2 \cdot ... \cdot a_n$ and they form $K = a_1 \cdot a_2 \cdot ... \cdot a_{n-1}$ groups.

The system's unit may be unable to operate either it is failed, or it is disconnected as a result of failure of some unit. The failure of (i, j) unit (i indicates the level, $i = \overline{0, n}$, j – the number of the unit of i-level, $j = \overline{1, N_i}$, $N_i = a_1 \cdot a_2 \cdot ... \cdot a_i$) leads to the disconnection of all units that are connected with this unit and are controlled by it and of all preceding units connected with it and that do not belong to any efficient way. We will understand here under an efficient way a chain of functional connected operating units from the principal $(0, 1)$ to one of the extreme.

The restored unit is included in system simultaneously with all previously disconnected operative units (with that level of efficiency at the moment when these units are disconnected) that form an efficient way with the restored unit. Moreover, the disconnected early units under repair continue (but not start again) their repair, if these units are functionally connected with the restored unit.

The system is considered in failure (total failure) if the number of efficient ways is less than R $(1 \leq R < N)$ and at this moment all operative units are disconnected.

It is assumed that

- the failure times $\alpha_1^{(ij)}$ and the repair times $\alpha_0^{(ij)}$ are independent in totality random variables with limited mean $0 < E\alpha_k^{(ij)} = T_k^{(ij)} < \infty$, $i = \overline{0, n}$, $j = \overline{1, N_i}$, $k = \overline{0, 1}$,
- the distribution functions of failure and repair times are considered absolutly continuous with respect to Lebesgue measure, $\overline{F}_k^{(ij)}(t) = 1 - F_k^{(ij)}(t) > 0$, $t > 0$,
- the restored unit is as good as new,
- there is no queue to repair,
- the disconnection and including of the units in system, as well as failure, are taking place instantaneously.

3 Semi-Markov Model

The functioning of such system is described (see [1]) by the semi-Markov processes $\xi(t) = \{\xi_{01}(t), \xi_{11}(t), \xi_{12}(t), ..., \xi_{21}(t), ..., \xi_{ij}(t), ..., \xi_{nN}(t); v_{01}(t), v_{11}(t), v_{12}(t), ..., v_{21}(t), ..., v_{ij}(t), ..., v_{nN}(t)\}$, where

$$\xi_{ij}(t) = \begin{cases} 1, & \text{if the } (i,j) \text{ unit is operative at the moment } t, \\ 0, & \text{if the } (i,j) \text{ unit is under repair at the moment } t. \end{cases}$$

$v_{ij}(t)$ – is (if $\xi_{ij}(t) = 0$) the repair time of (i,j) unit from its last failure and (if $\xi_{ij}(t) = 1$) the lifetime of (i,j) unit from its last join in the system (without taking in consideration the possible time of disconnection).

The phase space of system's states is $(Z, \mathcal{Z})$, where $Z = \{(d; x^{(ij)}) : d \in D, x^{(ij)} = (x_{01}, x_{11}, ..., x_{1N_1}, x_{21}, ..., x_{2N_2}, x_{i1}, ..., x_{ij-1}, 0, x_{ij+1}, ..., x_{nN}), x_{km} > 0, k = \overline{0,n}, m = \overline{1, N_k}, (k,m) \neq (i,j)\}$;
$D = \{d : d = (d_{01}, d_{11}, ..., d_{1N_1}, d_{21}, ..., d_{2N_2}, ..., d_{km}, ..., d_{nN}), d_{km} = \overline{0,1}, k = \overline{0,n}, m = \overline{1, N_k}\}$
x_{km} – points the time passed by the last change of "physical" state of the (k,m) unit, d_{km} – describes the "physical" state of the (k,m) unit:

$$d_{km} = \begin{cases} 1, & \text{if it is operative (or disconnected in an operative state)}, \\ 0, & \text{if it is under repair (or disconnected in an failured state)}, \end{cases}$$

$\mathcal{Z}$ – σ-algebra of Borel sets in Z.

We define the set of operative system's states Z_1 and the set of failure system's states Z_0 proceeding from the concept of total system's failure: $Z_1 = \{(d; x^{(ij)}) \in Z : d \in D_1\}$, $Z_0 = \{(d; x^{(ij)}) \in Z : d \in D_0\}$, where

$$D_1 = \{d \in D : \sum_{u=1}^{N} S_u \geq R\},\ D_0 = \{d \in D : \sum_{u=1}^{N} S_u < R\},$$

$$S_u = d_{nu} \cdot d_{n-1,u-1} \cdot ... \cdot d_{i,u_1} \cdot ... \cdot d_{1,u_1} \cdot d_{0,1},$$

$$u_i = \begin{cases} \left[\frac{u}{a_n \cdot a_{n-1} \cdot \ldots \cdot a_{i+1}}\right] + 1, & \text{if } u \neq 0(mod(a_n \cdot a_{n-1} \cdot \ldots \cdot a_i)), \\ \left[\frac{u}{a_n \cdot a_{n-1} \cdot \ldots \cdot a_{i+1}}\right], & \text{otherwise,} \end{cases}$$

where $[\cdot]$ denotes entire part of the number.

The mean life time T_1 of the system S is given by

$$T_1 = \left\{ \sum_{d\in D_1} \prod_{i=0}^{n} \prod_{j=1}^{N_i} T_{d_{ij}}^{(ij)} \right\} \cdot \left\{ \sum_{d\in D_0} \sum_{(i,j)\in I} \prod_{s=0}^{n} \prod_{\substack{v=1 \\ (s,v)\neq(i,j)}}^{N_s} T_{d_{sv}}^{(sv)} \right\}^{-1},$$

and the mean repair time T_0 of the system S is given by

$$T_0 = \left\{ \sum_{d\in D_0} \prod_{i=0}^{n} \prod_{j=1}^{N_i} T_{d_{ij}}^{(ij)} \right\} \cdot \left\{ \sum_{d\in D_0} \sum_{(i,j)\in I} \prod_{s=0}^{n} \prod_{\substack{v=1 \\ (s,v)\neq(i,j)}}^{N_s} T_{d_{sv}}^{(sv)} \right\}^{-1},$$

where I denotes the set of units under repair that are not disconnected at the state d.

4 Homogeneous Systems

Suppose now that the system S is homogeneous: the units of the i-level are of the same type $T_k^{(ij)} = T_k^{(i)}$, $k = \overline{0,1}$, $i = \overline{0,n}$. The system will be considered under total failure when the number of operative extreme groups is less the P $(1 \leq P < K)$. An extreme group is operative, if it contains Q or more operative units from a_n.

Then may be suggested an iterative algorithm for determining T_1 and T_0. For example, when $P = 1$

$$T_1 = \frac{T_1^{(0)} S_+(1)}{T_1^{(0)} S_*(1) + S_+(1)},$$

$$T_0 = \frac{T_0^{(0)} S_+(1) + T_1^{(0)} S_-(1)}{T_1^{(0)} S_*(1) + S_+(1)},$$

where

$$S_+(n-i) = \left[T_1^{(n-i)} S_+(n-i+1) + A_{n-i}\right]^{a_{n-i}} - S_-(n-i), i = \overline{1, n-1},$$

$$A_{n-i} = T_0^{(n-i)} S_+(n-i+1) + T_1^{(n-i)} S_-(n-i+1), i = \overline{1, n-1},$$

$$S_-(n-i) = [A_{n-1}]^{a_{n-i}}, i = \overline{1, n-1},$$

$$S_*(n-i) = a_{n-i} \left[S_+(n-i) + T_1^{(n-i)} S_*(n-i+1)\right] A_{n-i}^{a_{n-i}-1}, i = \overline{1, n-1},$$

$$S_+(n) = \left(T_1^{(n)} + T_0^{(n)}\right)^{a_n} - \sum_{k=a_n-Q+1}^{a_n} C_{a_n}^k \left(T_1^{(n)}\right)^{a_n-k} \left(T_0^{(n)}\right)^k;$$

$$S_-(n) = C_{a_n}^{a_n-Q+1} \left(T_1^{(n)}\right)^{Q-1} \left(T_0^{(n)}\right)^{a_n-Q+1};$$

$$S_*(n) = C_{a_n}^{a_n-Q+1} \left(T_1^{(n)}\right)^{Q-1} \left(T_0^{(n)}\right)^{a_n-Q}.$$

5 Conclusion

The main characteristics of the reliability of information-computer systems with hierarchical structure are obtained.

It should be mentioned that the results are obtained in terms of structure and means of failure and repair times and in a suitable for coding form.

References

[1] A. Corlat, V. Kuznetsov, M. Novicov, A. Turbin. *Semi-Markov models of systems with repair and queueing systems.* Shtiintsa, Kishinev, 1991 (in Russian).

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