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A new approach and software support to reliability analysis of repairable complex systems

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Abstract

A new method of reliability analysis of complex systems and software supporting is suggested. It is based on modelling of their evolution by means of semi–Markov processes. The results are obtained under general assumptions concerning the distributions of failure and repair times of the system's unit.

We will consider a complex system S with repairable units. It consists from N, $(N \geq 1)$ units and has a definite functional structure.

The failure time of the i–unit is a random variable (r.v.) $\alpha_1^{(i)}$ with the following distribution function (d.f.) $F_1^{(i)}(t) = P(\alpha_1^{(i)} \leq t)$, $i = \overline{1,N}$, and the repair time is a r.v. $\alpha_0^{(i)}$ with d.f. $F_0^{(i)}(t) = P(\alpha_0^{(i)} \leq t), i = \overline{1,N}$. It is assumed that d.f. $F_k^{(i)}$, $k = 0,1$, $i = \overline{1,N}$ are absolutely continuous with respect to Lebesgue measure, and r.v. $\alpha_k^{(i)}$ are independent in totality and have finite means $(0 \leq E\alpha_k^{(i)} < \infty)$

The i–unit may be in one of the following states: operative, operative disconnected, under repair or disconnected under repair.

The disconection of i–unit (or a totality of units) may be a result of total system's failure or of some functional connected with it system's unit and the i–unit does not belong to any efficient way. We will understand here under an efficient way a chain of functional connected units, whose functioning involves the system's viability. When the system's failure occurs all the remained operative units are disconnected. At this moment the repair of units under repair in system is done only.

The disconnected units are included in system with that level of operation or repair which find theirs at the moment of disconnection or of total system failure. These inclusions are taking place simultaneously with some restored unit in condition that these units generate an efficient way.

Moreover it is assumed that

– the restored unit is as good as new,

– there are not queuie to repair,

– the disconnection and including of the units in system as failure as are taking place instantaneously.

The concept of system's failure is introduce proceeding from its functional structure: it may be as a result of the failure of one or a group of units.

The i–system's unit functioning $i = \overline{1, N}$, represents a sequence of alternate periods of operating and repair (with posible disconnection) and the system's functioning — an analogue of superpossition of the N independent alternate renewal processes (see [2]).

Let $\xi_i(t) \quad i = \overline{1, N}$ — alternate processes with posible disconnection periods that model the functioning of i–unit with $P\{\xi_i(0) = 1\} = 1$ and $\xi_i(t) = 1$, if in the moment t the i–unit is operating or disconnected in an operative state, $\xi_i(t) = 0$ if it is under repair or desconnected under repair. Following the approuch suggested in [1] we will consider the semi–Markov processes $\xi(t) = \{\xi_1(t), \ldots, \xi_i(t), \ldots, \xi_N(t); u_1(t), \ldots, u_i(t), \ldots, u_N(t)\}$ that is modelling the system's evolution. Here $u_i(t)$ is the repair time of the i–unit from its last failure if $\xi_i(t) = 0$ and is the life time of the i–unit from its last join in system if $\xi_i(t) = 1$ (without taking in consideration the posible time of disconnection).

The phase space of system's states is $(Z, \mathcal{Z})$, where

$$
\begin{aligned}
Z &= \{z = (d; x^{(i)}) : d \in D,\ x^{(i)} \in R_+^{(i)}\}; \\
D &= \{d : d = (d_1, \ldots, d_i, \ldots, d_N),\ d_i = \overline{0, 1},\ i = \overline{1, N}\} \\
R_+^{(i)} &= \{x^{(i)} : x^{(i)} = (x_1, \ldots, x_{i-1}, 0, x_{i+1}, \ldots, x_N), \\
&\quad\ x_k > 0,\ k = \overline{1, N},\ k \neq i\}
\end{aligned}
$$

$\mathcal{Z}$ — σ–algebra of Borel sets in Z.

We will define the semi–Markov kernel of the processes $\xi(t)$

$$Q(t,(d;x^{(i)}),\{b;[0,y]^{(n)}\})$$
$$= \begin{cases} P\{x_n+\alpha_{d_i}^{(i)} > \alpha_{d_n}^{(n)}, \alpha_{d_j}^{(j)} > x_j+\alpha_{d_n}^{(n)}-x_n, x_j+\alpha_{d_n}^{(n)}-x_n \\ \in [0,y_j]; j \notin I_d, j\neq i, \alpha_{d_l}^{(l)} > x_i, l \in I_d, \alpha_{d_n}^{(n)}-x_n \\ \leq t \mid \alpha_{d_m}^{(m)} > x_m, m=\overline{1,N}\}, i\neq n; \\ P\{\alpha_{d_n}^{(n)} > x_n+\alpha_{d_i}^{(i)}, x_n+\alpha_{d_i}^{(i)} \in [0,y_n]; n \notin I_d, n \neq i; \\ \alpha_{d_l}^{(l)} > x_l, l\in I_d, \alpha_{d_i}^{(i)} \leq t \mid \alpha_{d_m}^{(m)} > x_m, m=\overline{1,N}\}, i=n; \end{cases} \quad (1)$$

where $I=\{i: d_i=0,\ i\notin I_d\}$; I_d — the set of indices of disconnected units in the state $(d;x^{(i)})$ $\{b;[0,y]^{(n)}\}=\{z\in Z: z=(b;y^{(n)}), y^{(n)} \in [0,y]^n\}$;
$[0,y]^{(n)}=([0,y_1],[0,y_2],\ldots,[0,y_{n-1}],0,[0,y_{n+1}],\ldots,[0,y_N])$;

$$[0,y_i]=\begin{cases}[0,x_i+y], & \text{if} \quad i\notin I_d, i\neq n;\\ [0,x_i], & \text{if} \quad i\in I_d, i\neq n;\end{cases}$$

The states d and b differs by the n–component only here. In other cases the transition probability from $(d;x^{(i)})$ to $\{b;[0,y]^{(n)}\}$ is zero.

Let us define the mean values of the being time of $\xi(t)$ in the states from Z

$$\Theta_{(d,x^{(l)})}=min\{[\xi_n^{(d_n)}-x_n]^+, n\notin I_d\}, \quad (2)$$

where $[\xi-x]^+$ r.v. with d.f. $P\{[\xi-x]^+\leq t\}=P\{\xi-x\leq t \mid \xi>x\}=[F(x+t)-F(x)]/\overline{F}(x)$.

The relations (1)–(2) (see [2]) completely define the semi–Markov processes $\xi(t)$ that model the evolution of the system S.

The mean life time T_1 of the system S is given by

$$T_1=\frac{\sum_{d\in D_1}\prod_{i=1}^N T_{d_i}^{(i)}}{\sum_{d\in D_0}\sum_{n\in I}\prod_{j=1 j\neq n}^N T_{d_j}^{(j)}}$$

and the mean repair time T_0 of the system S is given by

$$T_0=\frac{\sum_{d\in D_0}\prod_{i=1}^N T_{d_i}^{(i)}}{\sum_{d\in D_0}\sum_{n\in I}\prod_{j=1 j\neq n}^N T_{d_j}^{(j)}}$$

where

$$T_{d_i}^{(i)} = \begin{cases} E\alpha_1^{(i)}, & \text{if} \quad d_i = 1; \\ E\alpha_0^{(i)}, & \text{if} \quad d_i = 0; \end{cases}$$

D_1 (D_0) — is the set of operative (repair) states.

The described method of complex systems reliability determining needs a great number of mathematical operations. The necessity of the special software design appeared as a result.

The first version of our software (Reliability 1.0) ensures the determination of the reliability parameters for standard technical systems (series, parallel, parallel–series–parallel, bridges, hierarchical) with complex and simple structure, homogeneous and dissimilar elements, with the system deconnectation and without it. It also ensures the determination of the optimal structure for homogeneous parallel–series–parallel systems. A basis for calculation the reliability parameters of arbitrary systems was created simultaneously.

To make software usable for any user a developed graphics interface and large help and informational modules, which allow to obtain information about any appearing problem, were created. Informational and help modules can be desconnected on users desire.

Our software consists of main module, which unites graphics interface functions, input–output functions, calculating functions for classic systems, auxiliary functions, and two groups of files:

- data files for calculating reliability parameters of some systems;

- help and informational files (their presence is not obligatory for software work).

Graph–designer works separately. This module unites the simple graph editor and a set of functions which execute some algorithms on graphs and functions for reliability calculating in graph structures.

The presence of an IBM PC XT, AT, PS/2 or compatible with EGA or VGA display computer with DOS is necessary for software functioning. In addition , for graph–designer functioning the mouse presence is necessary.

We use an optimizing C compiler in order to obtain the compact and quick executive modules. Speed problem is essential, especially for arbitrary graph structures, where the algorithm for reliability parameters calculating has exponential complexity.

A software support may be used in reliability analysis of functioning technical systems and in design of new complex systems.

References

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