STUDY OF THE INCLINOMETER CALIBRATION METHOD USING GEODETIC MEASUREMENTS

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Abstract: *The inclinometer is used for small-angle measurement and can be fixed on the radio-communication tower to obtain changes relative to the construction's vertical plane. The real inclination of the tower can be calculated by modelling the variables obtained from the inclinometer in combination with geodetic measurements. In order to perform assembly error calibration a precise mathematical model based on inclinometer data and total station measurements is proposed. For experimental measurements a dual axis inclinometer and a reference prisms installed on the rigid platform were used. Finally, through simulations of various inclination of the platform, the calibration method is validated and the inclinometer measurement accuracy is evaluated. The preliminary results show the possibility to use this calibration method in order to determine initial position of the inclinometer installed on the tower construction.*

Keywords: *tower construction, inclinometer calibration, geodetic measurement, total station.*

1. Introduction

The worldwide ongoing process of using microelectromechanical systems (MEMS) technology for almost every possible sensing modality assume production of variety of sensors for measuring pressure, motion, acceleration, temperature, magnetic field, and light, as well as gyroscopes, inclinometers, switches, capacitive touch sensors, etc [5].

This paper is focused on using MEMS dual axis inclinometer for small-angles measurement on the radio-communication tower to obtain changes relative to the construction's vertical axis. However, the real inclination of the tower can be calculated by modelling the variables obtained from the inclinometer data in combination with geodetic measurements.

In most cases it is very difficult to install inclinometer sensor on the metal detales of the tower construction exactly in the horizontal position perpendicular to the plumb line. In order to calibrate the inclinometer on the tower it is necessary to know the initial inclination of the construction defined as the angle between the gravity vector and vertical axis of the construction.

There are different methods of tower verticality determination using Global Navigation Satellite Systems (GNSS) observations, three-dimensional terrestrial geodetic measurements using total station or traditional geodetic measurements methods well described by different authors [1-3]. In this paper, a precise mathematical model based on inclinometer data and total station measurements is proposed in order to perform assembly calibration errors. Also the accuracy of proposed inclinometers was estimated using the high precision dual axis inclinometer as standard for calibration.

2. Frame systems and orientation parameters

A total of three different topocentric coordinate systems are defined (see figure 1).

Total station topocentric coordinate system with origin in the intersection of instrument vertical axis U_{TS} with telescope axis and N_{TS}, E_{TS} axis oriented to the North and East directions respectively.

Topocentric coordinate system with origin in the in the centre of prism P1 and N'_{TS}, E'_{TS} U'_{TS} axis oriented parallel to the total station topocentric coordinate system.

Sensor platform coordinate system with origin in the centre of prism P1, U_{PT} axis oriented parallel to total station topocentric coordinate system and N_{PT} axis oriented to the prism P_2 by yaw angle α from N_{TS} axis and E_{PT} rotated 90 degrees clock verso from the N_{PT} axis.

Inclined sensor platform coordinate system by pitch angle β rotated clock verso around axis N'_{PT} with origin in the centre of prism P_1 and E'_{PT}, U'_{PT} respectively.

Inclined sensor platform coordinate system by roll angle γ rotated clock wise around axis E''_{PT} with origin in the centre of prism P_1 and N''_{PT}, U''_{PT} respectively.

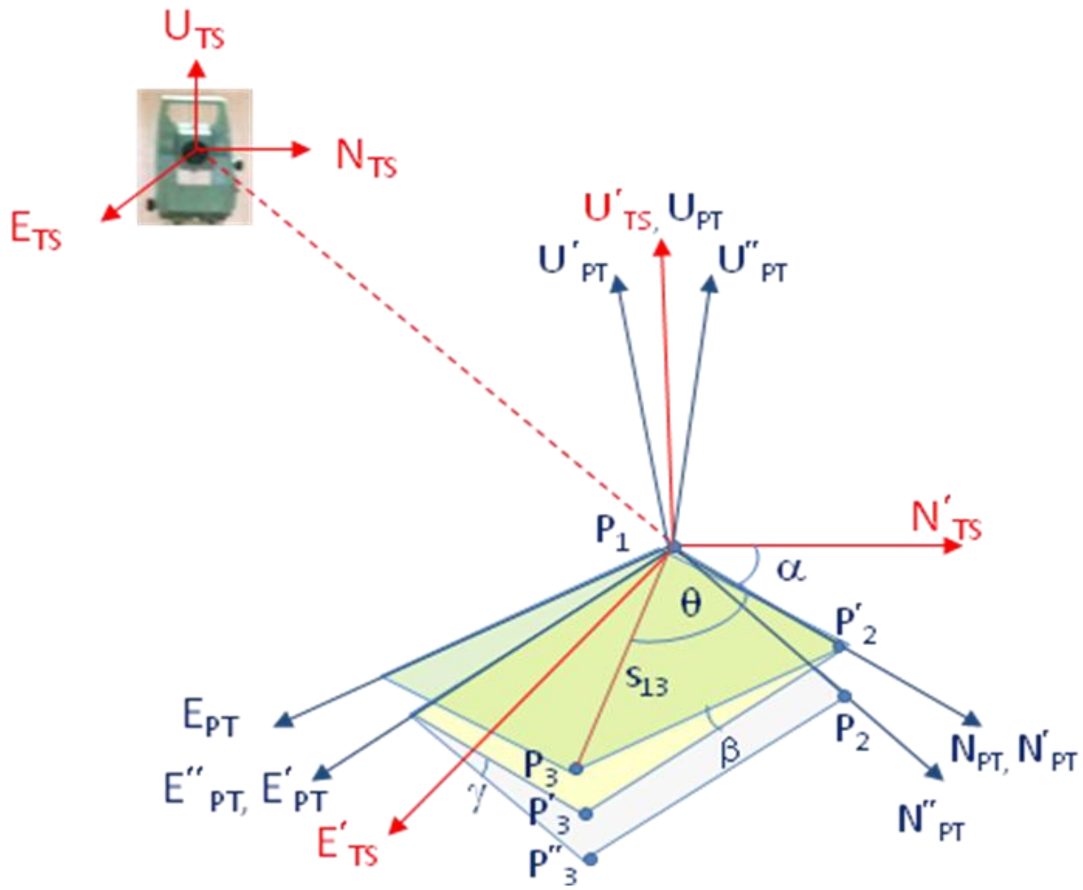

Fig.1. Relationship between topocentric coordinates and sensor platform orientation parameters.

Relationship between geodetic topocentric coordinates N'_{TS}, E'_{TS}, U'_{TS}, and sensor platform system of coordinates N''_{PT}, E''_{PT}, U''_{PT} are defined by rotation matrix R [4]:

$$R = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\gamma \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}. \tag{1}$$

Geodetic topocentric coordinates N'_{TS}, E'_{TS}, U'_{TS}, could be determined by using translation vector $[\Delta N, \Delta E, \Delta U]^T$ from total station system of coordinate to sensor platform system of coordinate:

$$\begin{bmatrix} N'_{TS} \\ E'_{TS} \\ U'_{TS} \end{bmatrix} = \begin{bmatrix} N_{TS} \\ E_{TS} \\ U_{TS} \end{bmatrix} + \begin{bmatrix} \Delta N \\ \Delta E \\ \Delta U \end{bmatrix}. \tag{2}$$

Relationship between orientation angle (yaw) α, vertical angle *z* , distance *s* and total station system of coordinates N_{TS}, E_{TS} ,U_{TS} are well known as the following [4]:

$$\begin{bmatrix} N_{TS} \\ E_{TS} \\ U_{TS} \end{bmatrix} = s \begin{bmatrix} \sin z \cos\alpha \\ \sin z \sin\alpha \\ \cos z \end{bmatrix}; \quad \alpha = arctg\,\frac{E}{N}; \quad z = arctg\,\frac{\sqrt{N^2+E^2}}{U}; \quad s = \sqrt{N^2+E^2+U^2}. \tag{3}$$

The equations (1-3) are the foundation for development of measurement model for determination of orientation parameters.

3. Measurement model for orientation parameters determination

In order to derive parameters yaw-pitch-roll from total station measurements on the orientation angle α, vertical angle *z* and distance *s*, the inclination of sensor platform was simulated using iron plate installed on tribrach with 3 fixed mini-prisms as showed in fig. 2. For demonstration on the sensor platform were fixed two dual-axis Bewis Sensing inclinometers with accuracy 0.01° for calibration and 0.001° as standard for measuring pitch and roll angels.

The measurements were done by Leica Geosystems total station TC802 with accuracy $m_\alpha = \pm 2''$ and $m_s = \pm(2+2\text{ppm})$ mm in 14 sessions with sensor platform inclinations angles from 0 to -3 degrees.

The translation vector was determined as differences of topocentric coordinates of prism P3 and prism P1 in order to identify origin and the orientation angle α of platform coordinate system:

$$\begin{bmatrix} N'_{TS} \\ E'_{TS} \\ U'_{TS} \end{bmatrix}_{P2} = \begin{bmatrix} N_{TS} \\ E_{TS} \\ U_{TS} \end{bmatrix}_{P2} - \begin{bmatrix} N_{TS} \\ E_{TS} \\ U_{TS} \end{bmatrix}_{P1}, \qquad \alpha = arctg\,\frac{E_{TS2}}{N_{TS2}}; \tag{4}$$

Once the orientation angle α is obtained as in equations (4) a sensor platform could be oriented by rotation matrix around prism P_1 and the pitch angle along the axis N_{PT} would be determined using the following formulas:

$$\begin{bmatrix} N_{PT} \\ E_{PT} \\ U_{PT} \end{bmatrix}_{P2} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} N'_{TS} \\ E'_{TS} \\ U'_{TS} \end{bmatrix}_{P2}, \qquad \beta = arctg\,\frac{U_{PT2}}{N_{PT2}}. \tag{5}$$

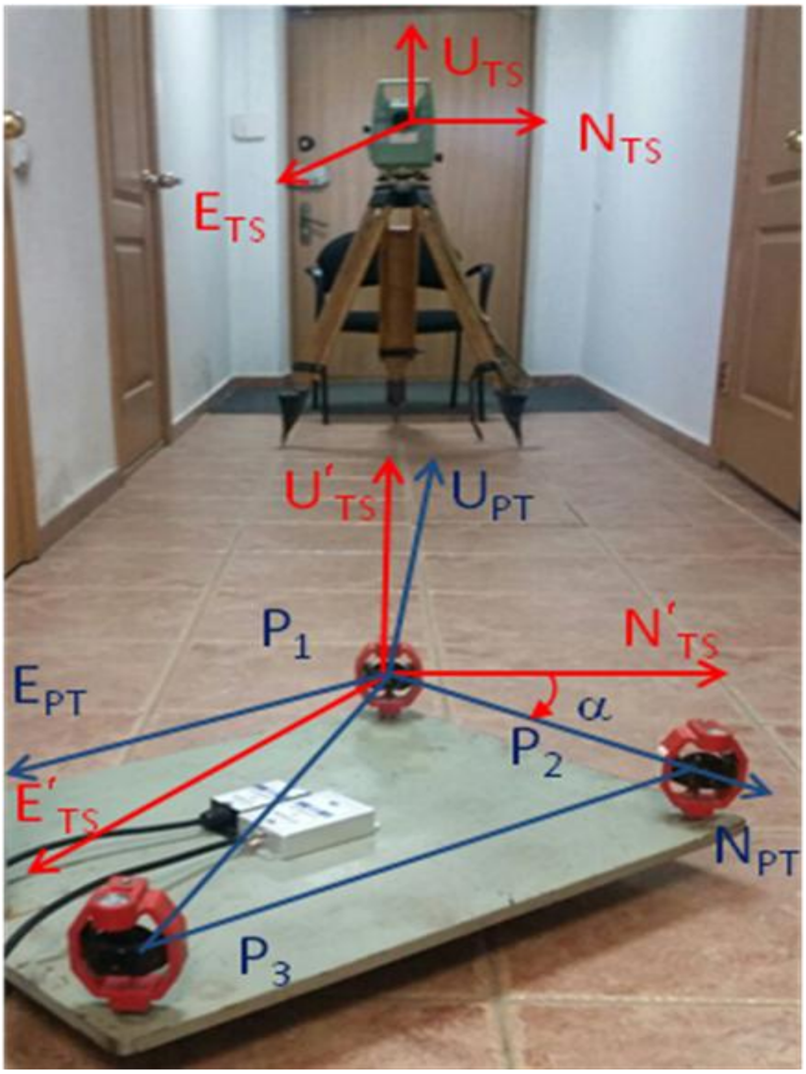

Fig. 2. The demonstration sensor platform for measuring pitch and roll angels

Once the estimates of orientation α and pitch β angles are obtained as in equations (4-5), a series of rotations needs to be carried out prior to determining the roll angle γ from the measurements to prism P_3 [2]:

$$\begin{bmatrix} N'_{TS} \\ E'_{TS} \\ U'_{TS} \end{bmatrix}_{P3} = \begin{bmatrix} N_{TS} \\ E_{TS} \\ U_{TS} \end{bmatrix}_{P3} - \begin{bmatrix} N_{TS} \\ E_{TS} \\ U_{TS} \end{bmatrix}_{P1}, \quad \begin{bmatrix} N_{PT} \\ E_{PT} \\ U_{PT} \end{bmatrix}_{P3} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} N'_{TS} \\ E'_{TS} \\ U'_{TS} \end{bmatrix}_{P3}, \tag{6}$$

$$\begin{bmatrix} N'_{PT} \\ E'_{PT} \\ U'_{PT} \end{bmatrix}_{P3} = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\gamma \end{bmatrix} \times \begin{bmatrix} N_{PT} \\ E_{PT} \\ U_{PT} \end{bmatrix}_{P3}, \quad \begin{bmatrix} N''_{PT} \\ E''_{PT} \\ U''_{PT} \end{bmatrix}_{P3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix} \times \begin{bmatrix} N'_{PT} \\ E'_{PT} \\ U'_{PT} \end{bmatrix}_{P3}. \tag{7}$$

Taking in account that a third rotation by roll angle γ, rotates the prism P3 coordinates $N'_{PT}, E'_{PT}, U'_{PT}$ to its platform frame $U''_{PT} = 0$. From last row the roll angle γ could be derived as following:

$$\sin\gamma \cdot E'_{PT3} + \cos\gamma \cdot U'_{PT3} = 0, \quad \text{therefore,} \quad \gamma = arctg\frac{U'_{PT2}}{E_{PT2}}. \tag{8}$$

Coordinates $N''_{PT}, E''_{PT}, U''_{PT}$ can be calculated also for control using distance S_{13} and angle θ between axis N_{PT} and direction from prism P_1 to prism P_3:

$$N''_{PT3} = S_{13} \cdot \cos\theta, \quad E''_{PT3} = S_{13} \cdot \sin\theta, \quad \text{were} \tag{9}$$

$$S_{13} = \left[(U_{TS3} - E_{TS1})^2 + (U_{TS3} - E_{TS1})^2 + (U_{TS3} - E_{TS1})^2\right]^{\frac{1}{2}}.$$

Finally, through simulations of various inclination of the sensor platform, the calibration method is validated and the inclinometer measurement accuracy is evaluated (see table 1).

Table 1. Comparison of measured and calculated pitch and roll and determination of calibration parameters (average) and they standard deviation (STD)

Session number	**Measured pitch ($\beta°_m$)**	**Measured roll ($\gamma°_m$)**	**Calculated pitch ($\beta°_c$)**	**Differences $\Delta\beta=\beta_c-\beta°_m$**	**Calculated roll ($\gamma°_c$)**	**Differences $\Delta\beta=\beta_c-\beta°_m$**
1	-1.1510	-0.1908	1.1654	0.0308	-0.3377	-0.1469
2	-1.1764	-0.0748	-1.0890	0.0874	-0.3516	-0.2768
3	-1.1792	-0.0648	-1.1052	0.0740	-0.3094	-0.2446
4	-0.5362	-3.0214	-0.3759	0.1603	-3.2435	-0.2221
5	-0.8932	-1.3802	-0.7879	0.1053	-1.6026	-0.2224
6	-1.3868	-1.8156	-1.2986	0.0882	-2.0109	-0.1953
7	-0.8194	-1.9860	-0.6762	0.1432	-2.2158	-0.2298
8	-0.5306	-2.0796	-0.4531	0.0775	-2.2689	-0.1893
9	-1.6774	-1.7038	-1.5512	0.1262	-1.9272	-0.2234
10	-0.8438	-0.9646	-0.7829	0.0609	-1.1375	-0.1729
11	-1.0758	0.1326	-0.9963	0.0795	-0.1705	-0.3031
12	-0.8512	-0.9338	-0.7566	0.0946	-1.1517	-0.2179
13	-0.6930	-1.6650	-0.5658	0.1272	-1.9059	-0.2409
14	-0.5096	-2.5366	-0.3952	0.1144	-2.7294	-0.1928
			STD =	0.0343	STD =	0.0403
			Average =	0.0978	Average =	-0.2199

The results of comparison of calculated from total station measurements and measured by inclinometer with accuracy 0.01 degrees are visualized using diagrams in figures 4,5.

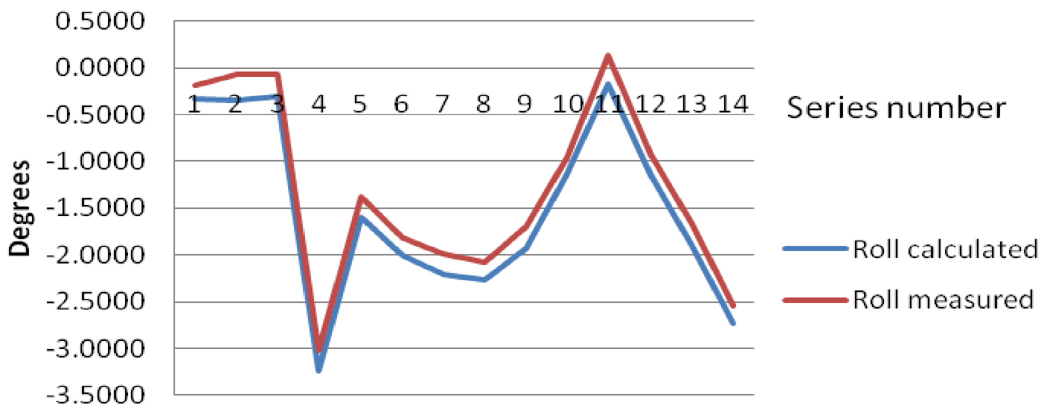

Fig. 3. Comparison of calculated and measured by inclinometer roll angle

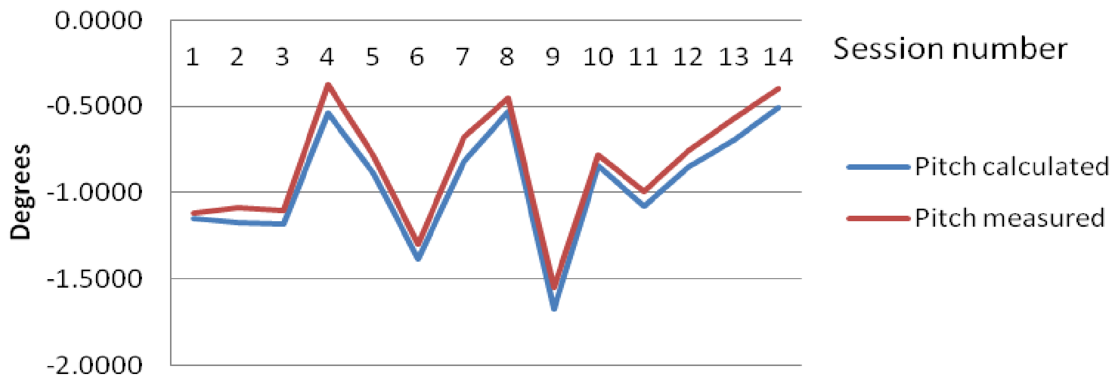

Fig. 4. Comparison of calculated and measured by inclinometer pitch angle

The results show the possibility to use this calibration method in order to determine initial position of the inclinometer installed on the tower construction.

4. Accuracy of orientation parameters determination

The accuracy of the computed pitch and roll by direct computation formulas can be derived based on laws of error propagation [2]:

$$m_\beta = \frac{m_U}{S}, \qquad m_\gamma = \frac{m_{NE}}{S \cdot \cos\theta}. \tag{10}$$

The influence of total station coordinate measurements with 2.5 mm accuracy on pitch and roll angles errors function on distances between prisms are shown on figure 5.

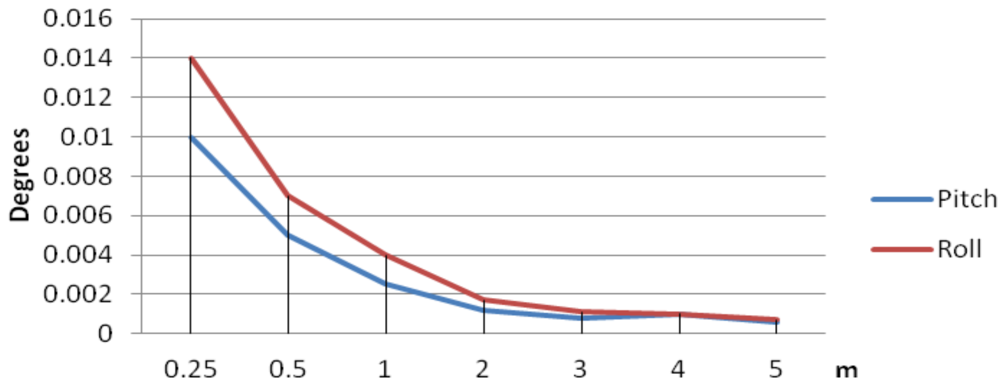

Fig. 5. The influence of total station coordinate measurements on pitch and roll angles

From equation (10) results that the accuracy of pitch and roll are inverse proportional to the distance between prisms.

The results of demonstration test shows the standard deviation of calculated pitch and roll angles 0.034° and 0.040° respectively (see table 1). The accuracy estimation using standard dual axis inclinometer shows better results of pitch and roll measurements 0.007° and 0.004°.These differences could be interpreted as influence of floor vibrations because of indoor measurements.

The average of differences between measured and calculated pith and roll angles could be considered as corrections in inclinometers installed on the radio-communication tower at different levels.

5. Conclusions

The agreement between inclinometer measurements and independent geodetic methods of computing yaw-pith-roll parameters shows the necessity to continue investigations on the radio-communication tower constructions using different inclinometers and geodetic equipment.

The results of investigations shows the necessity to use high precision total stations at least 2 arcsec and 2mm+2ppm precision total station in order to calibrate inclinometer sensors on the tower construction with a level of precision comparable to the angular accuracy achievable by 0.01 degrees precision inclinometers.

In conclusion, the geodetic methods of determination of yaw-pitch-roll parameters should be used for installation of inclinometer sensors in order to avoid any influence of construction details due to the irregularity of reference surface.

The simulation examples of the influence of total station accuracy on determination of yaw-pitch-roll parameters show necessity to take in consideration possibility to increase distances between prisms for high precision calibration of inclinometer sensors.

The future investigations should be oriented to combinations of total station measurements with GNSS observations in order to increase the accuracy of installation of inclinometer sensors.

6. References

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