# Height Reference Surface for high accuracy Digital Elevation Modelling in Republic of Moldova

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#### **Abstract**

In 2011 Laser Scanning of flood risk areas was done by Land Relation and Cadastre Agency in the frame of the project financed by Norwegian Government. The future plans are to cover all territory of the country with LiDAR data. However, as the precision and availability of LiDAR elevation data has improved the need to tie the LiDAR elevations to a National Levelling Network has increased. When combined with GNSS and inertial navigation data the ranges stored by the onboard computer may be converted to a 3-dimensional spot position - in this case expressed in Terrestrial European Reference System (ETRS89) coordinates based on GRS80 ellipsoid. To utilize this data at local or national level it is necessary to convert the ellipsoidal heights to a vertical datum such as the Baltic Sea 1977. This conversion requires the use of a high accuracy Height Reference Surface (HRS) that may be developed by conducting GNSS measurements on the national levelling network benchmarks and using Earth Gravitational Model EGM2008. This paper presents the results of comparative study of Height Reference Surface modelling methods using control GNSS/Levelling measurements and EGM2008 model. The GNSS measurements were carried out on 40 first and second order levelling benchmarks of National Levelling Network. For HRS determination three, four, five and seven parameters equations and multi-quadratic surface methods were used. Comparative analyses of examined methods showed the increasing accuracy from 5.6 cm using parameters equations to 4.6 cm with multi-quadratic surface model that correspond to fourth order levelling and could be used for high resolution Digital Elevation Modelling for all territory of the country. The analyses of residuals showed the best results in the middle part country territory and the worse on the eastern and western borders were is a lack of high quality data.

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### Keywords

Quasigeoid, Height Reference Surface, GNSS/Levelling, parameters equation, multi-quadratic surface.

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#### 1. Introduction

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Acceleration of LiDAR technologies increases the accuracy of heights determination. Obtained from GNSS and inertial navigation data measurements heights are geodetic (ellipsoidal) and for practical use needs to be converted into normal heights, as a distance between point on the physical surface and quasigeoid surface at gravity vector direction.

Determination of normal heights from ellipsoidal height depends on the accuracy of GNSS measurements, inertial navigation data and height anomalies calculated from global, regional or local quasigeoid model as a distance between ellipsoid and quasigeoid surface along the gravity vector.

First quasigeoid model GM2005 for Republic of Moldova territory was calculated in 2005 by Ukrainian Research Institute of Geodesy and Cartography, using 803 GNSS/levelling sites and European Geoid Model EGG97, transformed to Baltic Sea 1977 normal height system [1].

In order to develop high accuracy local gravimetric geoid in 2006 Institute of Geodesy, Engineering Research and Cadastre INGEOCAD subordinated to The Land Relations and Cadastre Agency of Republic of Moldova in cooperation with the National Geospatial-Intelligence Agency (NGA) of United States of America performed gravity campaign to establish first order National Gravity Network with accuracy 10  $\mu$ Gal using three LaCoste & Romberg G meters (Fig.1). In order to constrain the relative gravity measurements 3 absolute gravity stations were determined with accuracy 5  $\mu$ Gal, using FG5 absolute gravimeter [2].

The second and third order National Gravity Network (Fig.1) with density of 1 gravity point per 15-20 square km was performed in 2007-2008 by Institute of Geodesy, Engineering Research and Cadastre (INGEOCAD) with accuracy 20  $\mu$ Gal and 40  $\mu$ Gal respectively [3].

In 2012 a new gravimetric quasigeoid model GM2012 was determined by applying the Least Squares Modification of Stokes' formula with Additive corrections (LSMSA), also called the KTH method [4].

To generate and distribute height anomalies for real time normal height determination from GNSS measurements using MOLDPOS service, height reference surface based on precise GNSS/levelling was calculated by Technical University of Moldova in cooperation with Karlsruhe University of Applied

Science. The quasigeoid model GM2010 was calculated using HSKA method for digital finite element Height Reference Surface representation as polynomial solution [5].

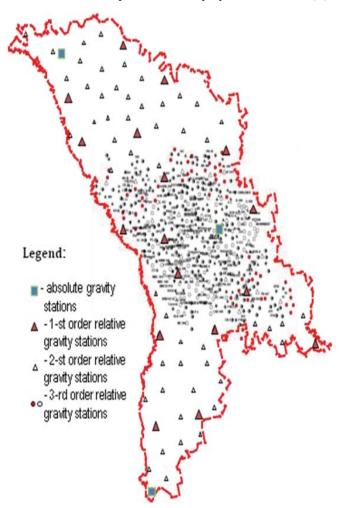


Fig. 1 Developed National Gravity Network

In 2014 the Land Relations and Cadastre Agency performed GNSS measurements on 1-st and 2-nd order levelling network benchmarks in order to estimate the accuracy of existing quasigeoid models for the Republic of Moldova territory [6].

Thanking in account that part of gravity data of Republic of Moldova territory were involved in calculation of EGM2008 model, Land Relation and Cadastre Agency decided to adopt it at the national level.

This study presents the results of comparative analyzes of Height Reference Surface modelling methods using control GNSS/Levelling measurements and EGM2008 model.

## 2. Height Reference Surface modelling methods

For HRS modelling a well know geometrical equations were used:

$$h - H - \zeta_i^{Model} = 0, \tag{1}$$

$$\Delta \zeta_{i} = \zeta_{i}^{GNSS/lev.} - \zeta_{i}^{Model} = h_{i} - H_{i} - \zeta_{i}^{Model}, \quad (2)$$

where  $\Delta \zeta_i$  is a height anomalies differences calculated between height anomalies  $\zeta_i^{GNSS/lev}$  and  $\zeta_i^{Model}$  determined from quasigeoid models, or differences between ellipsoidal height  $h_i$ , normal height  $H_i$  and height anomalies  $\zeta_i^{Model}$ calculated from quasigeoid models, where i = 1, 2,...,n and n is the number of observations.

In practice these equations are never satisfied due to a number of factors like random errors of measurements and approximations, datum and deformations which can affect the height systems, datum and benchmarks.

The parametric observations model could be written as following:

$$\Delta \zeta_i = \mathbf{a}_i \mathbf{x} + \mathbf{v}_i, \tag{3}$$

where  $\mathbf{x}$  are unknown parameters,  $\mathbf{v_i}$  are residuals and  $\mathbf{a_i}$ are observation coefficients corresponding to the number of parameters [7]:

for the 3 parameter model

$$\mathbf{a_i x} = (\cos \phi_i \cos \lambda_i) x_1 + (\cos \phi_i \sin \lambda_i) x_2 + (\sin \phi_i) x_3, (4)$$

whereas the 4-parameter model is given by

$$\mathbf{a_i x} = (\cos \phi_i \cos \lambda_i) x_1 + (\cos \phi_i \sin \lambda_i) x_2 + (\sin \phi_i) x_3 + (\sin \phi_i) x_4,$$
(5)

the 5-parameter model is

$$\mathbf{a}_{i}\mathbf{x} = (\cos\phi_{i}\cos\lambda_{i})x_{1} + (\cos\phi_{i}\sin\lambda_{i})x_{2} + (\sin\phi_{i})x_{3} + (\sin^{2}\phi_{i})x_{4} + x_{5},$$
(6)

the 7 parameter model is

$$\begin{aligned} \mathbf{a_i x} &= (\cos \phi_i \cos \lambda_i) x_1 + (\cos \phi_i \sin \lambda_i) x_2 + (\sin \phi_i) x_3 + \\ \frac{\cos \phi_i \cos \lambda_i \sin \phi_i}{\sqrt{1 - e^2 \sin^2 \phi_i}} x_4 + \frac{\cos \phi_i \sin \lambda_i \sin \phi_i}{\sqrt{1 - e^2 \sin^2 \phi_i}} x_5 + \\ \frac{\sin^2 \phi_i}{\sqrt{1 - e^2 \sin^2 \phi_i}} x_6 + x_7, \end{aligned}$$

while the multi-quadratic surface model is [8]

$$\mathbf{a}_{i}\mathbf{x} = \sum_{j=1}^{n} x_{i} a_{j}(\phi_{i}, \lambda_{i}, \phi_{j}, \lambda_{j}), \tag{8}$$

(7)

were  $x_i$  are surface coefficients and  $a_{ij}$  are distances calculated from point i to each point j [8].

The matrix form for the system of observation equations follows:

$$\Delta \zeta = \mathbf{A}\mathbf{x} + \mathbf{v},\tag{9}$$

where x is a vector of unknown parameters, v is vector of residuals and A is the design matrix that contains for each observation coefficients.

The parameters were estimated by least squares method:

$$\mathbf{x} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{\Delta} \zeta. \tag{10}$$

Introducing the vector of estimated parameters  $\mathbf{x}$  into the system of observation equations, is obtained the vector of estimated residuals

$$\mathbf{v} = \mathbf{\Delta}\boldsymbol{\zeta} - \mathbf{A}\mathbf{x}.\tag{11}$$

The estimated residuals  ${\bf v}$  are used to calculate the standard deviation:

$$\sigma_0 = \sqrt{\frac{\mathbf{v}^{\mathsf{T}} \mathbf{v}}{n - m}},\tag{12}$$

where n is the number of GNSS/levelling observations and m is the number of estimated parameters.

## 3. Accuracy evaluation of HRS models

For accuracy evaluation of HRS models were selected 40 first and second order levelling benchmarks of National Levelling Network (Fig 2).

The height anomalies from GNSS measurements on levelling benchmarks  $\zeta_i^{GNSS/lev}$  were determined in 2014 by INGEOCAD specialists, using two frequency GNSS receivers, in static mode, with duration 90 min and postprocessed with a connection to MoldPos GNSS Network.

The heights anomalies  $\zeta_i^{EGM2008}$  were calculated for each levelling benchmark using EGM2008 harmonic coefficients to degree and order 2159.

The height anomalies differences calculated between height anomalies  $\Delta \zeta_i$  were determined as following:

$$\Delta \zeta_{\rm i} = \zeta_{\rm i}^{\rm GNSS/lev.} - \zeta_{\rm i}^{\rm EGM\,2008}, \tag{13}$$

were i is number of GNSS/Levelling measurements.

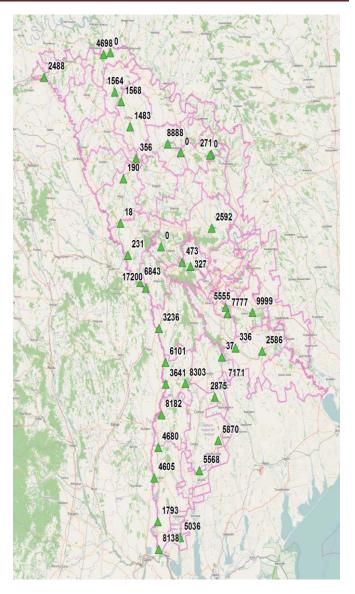


Fig. 2 GNSS/Levelling measurements

The preliminary estimation of quasigeoid models accuracy was done by INGEOCAD [7]. The results of calculations without correction surface (0P) are included in Table 1. The results of HRS accuracy estimation using the 3, 4, 5,7 parameters equations (3P, 4P, 5P, 7P) and multi-quadratic surface model (MQ) are shown in Table 1.

**Table 1** The results of estimation of HRS models accuracy

Statistics	Min. (m)	Max.(m)	Mean (m)	$\sigma_0(m)$
0P	-0.124	0.195	-0.007	0.058
3P	-0.101	0.204	0.000	0.055
4P	-0.102	0.202	0.000	0.056
5P	-0.102	0.203	0.000	0.057
7P	-0.087	0.199	0.000	0.055
MQ	-0.101	0.064	0.005	0.046

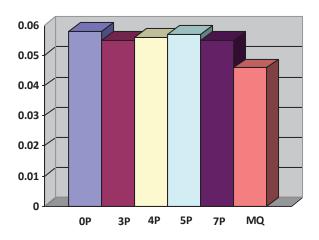


Fig. 3 Diagram of HRS models accuracy comparison

The HRS models standard deviations using different methods are shown in Figure 3. Comparative analyses of examined methods showed the increasing accuracy from 5.5 cm using 7 parameters equations to 4.6 cm with multiquadratic surface model.

#### 4. Conclusions

Comparative study of different methods of HRS modelling shows the best accuracy 4.6 cm of multi-quadratic surface model that correspond to fourth order levelling and could be used for high resolution Digital Elevation Modelling for all territory of the country. The parameter equations are recommended to use for calculation of HRS trend value.

The analyses of residuals showed the best results in the middle part country territory and the worse on the eastern and western borders were is a lack of high quality data.

For future improvement of HRS model a fitting GNSS/Leveling points related to 1st, 2nd order leveling networks, and carefully selected 3rd and 4th order leveling benchmarks have to be used taking in account weights corresponding to the accuracy of leveling benchmarks determination.

Improvement of HRS accuracy could be done by development of regional gravity geoid in cooperation with Romanian and Ukrainian national mapping and cadastre agencies signing gravity data exchange agreements.

The results of this study could be used by Land Relation and Cadastre Agency for improvement of existing Digital Elevation Models and processing of future LiDAR data using a new HRS model.

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