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Nambu-Goldstone modes of the two-dimensional Bose-Einstein condensed magnetoexcitons

S.A. Moskalenko[1], M.A. Liberman[2], D.W. Snoke[3], E.V. Dumanov[1,a], S.S. Rusu[1], and F. Cerbu[1]

[1] Institute of Applied Physics of the Academy of Sciences of Moldova, Academic Str. 5, 2028 Chisinau, Republic of Moldova
[2] Department of Physics, Uppsala University, Box 530, 75121 Uppsala, Sweden
[3] Department of Physics and Astronomy, University of Pittsburgh, 3941 O'Hara Street, 15260 Pittsburgh, USA

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Abstract. The collective elementary excitations of two-dimensional magnetoexcitons in a Bose-Einstein condensate (BEC) with wave vector $\boldsymbol{k} = 0$ were investigated in the framework of the Bogoliubov theory of quasiaverages. The Hamiltonian of the electrons and holes lying in the lowest Landau levels (LLLs) contains supplementary interactions due to virtual quantum transitions of the particles to the excited Landau levels (ELLs) and back. As a result, the interaction between the magnetoexcitons with $\boldsymbol{k} = 0$ does not vanish and their BEC becomes stable. The equations of motion for the exciton operators $d(P)$ and $d^\dagger(P)$ are interconnected with equations of motion for the density operators $\rho(P)$ and $D(P)$. Instead of a set of two equations of motion, as in the case of usual Bose gas, corresponding to normal and abnormal Green's functions, we have a set of four equations of motion. This means we have to deal simultaneously with four branches of the energy spectrum, the two supplementary branches being the optical plasmon branch represented by the operator $\rho(P)$ and the acoustical plasmon branch represented by the operator $D(P)$. The perturbation theory on the small parameter $v^2(1 - v^2)$, where v^2 is the filling factor and $(1 - v^2)$ is the phase space filling factor was developed. The energy spectrum contains only one gapless, true Nambu-Goldstone (NG) mode of the second kind with dependence $\omega(k) \approx k^2$ at small values k describing the optical-plasmon-type oscillations. There are two exciton-type branches corresponding to normal and abnormal Green's functions. Both modes are gapped with roton-type segments at intermediary values of the wave vectors and can be named as quasi-NG modes. The fourth branch is the acoustical plasmon-type mode with absolute instability in the region of small and intermediary values of the wave vectors. All branches have a saturation-type dependencies at great values of the wave vectors. The number and the kind of the true NG modes is in accordance with the number of the broken symmetry operators. The comparison of the results concerning two Bose-Einstein condensates namely of the coplanar magnetoexcitons and of the quantum Hall excitons in the bilayer electron systems reveals their similarity.

1 Introduction

A two-dimensional electron system in a strong perpendicular magnetic field reveals fascinating phenomena such as the integer and fractional quantum Hall effects [1–5]. The discovery of the fractional quantum Hall effect (FQHE) fundamentally changed the established concepts about charged single-particle elementary excitations in solids [6–8]. The gauge transformations from the wave functions and creation operators of simple particles to another wave functions and creation operators describe composite particles (CPs) [9–11]. They are made from the previous particles and quantum vortices, created under the influence of the magnetic flux quanta [12–21]. Due to the contribution of many outstanding investigations [1–28] as well as many efforts to explain and to represent the underlying processes in a more clear way [4,29,30] it is possible to make a short summary as follows. One can begin with the concept of composite particles proposed by Wilczek [9,10] in the form of particles with magnetic flux tubes attached. A posteriori, the flux tubes were substituted by quantum vortices, as was argued by Read in a series of papers [12–15]. In many explanations proposed by Enger [4], it was underlined that in 3D space the particles may obey only Fermi and Bose statistics, whereas in 2D space the fractional statistics are also possible. Now under the interchanging of two particles, the wave function obtains the phase factor $e^{i\pi\alpha}$ with any fractional values of α. Such particles were named "anyons" [9,10]. The gauge transformations [29] of the wave functions and of the creation and annihilation operators of the initial particles and the corresponding Hamiltonians was a powerful instrument revealing the fundamental physical processes hidden at the first sight in the quantum states of the system.

[a] e-mail: dum@phys.asm.md

The gauge transformation revealed the existence of the vortices in the system created by the magnetic flux quanta.

Girvin et al. [20,21] elaborated the theory of the collective elementary excitation spectrum in the case of the FQHE, closely analogous to Feynman's theory of superfluid helium. The predicted spectrum has a gap at $k = 0$ and a deep magneto-roton minimum at finite wavevector, which is a precursor to the gap collapse associated with Wigner crystal instability.

In this paper we study a coplanar electron-hole (e-h) system with electrons in a conduction band and holes in a valence band, both of which have Landau levels in a strong perpendicular magnetic field. Earlier, this system was studied in a series of papers mostly dedicated to the theory of 2D magnetoexcitons [31–43]. This system bears some resemblance to the case of a bilayer electron system [44–49]. A short review concerning the Bose-Einstein condensation (BEC) of the quantum Hall excitons (QHExs) arising in the bilayer electron systems in the conditions of the quantum Hall effect (QHE) at one half filling factor $\nu = 1/2$ for each layer and the total filling factor for two layer equal to unity $\nu_t = 1$ is needed. The aim of the review is to compare this phenomenon with the case of BEC of two-dimensional (2D) magnetoexcitons. Such comparison will permit to better understand the underlying physics and to verify the accuracy of the made approximations. In reference [44] Fertig investigated the energy spectrum of a bilayer electron systems in a strong perpendicular magnetic field and introduced the concept of the interlayer phase coherence of the electron states in two adjacent layers, which leads to the model of quantum Hall excitons in condition of their BEC. Unexpectedly a strong evidence of exciton BEC was ultimately found in a such surprising place as a double layer 2D electron system at high magnetic field [45]. In the QHE regime the excitons consist from electrons in the lowest Landau level (LLL) of the conduction band of one layer being bound to the holes which appear in the LLL of the conduction band in another layer. The ground state wave function proposed by Fertig [44] introduces the interlayer phase coherence reflecting a new state, in which the electrons are no-longer confined to one layer or to another, but instead of it they reside in coherent liniar combinations of the two layer states as follows

$$|\psi\rangle = \prod_t \left(ua_{1t}^\dagger + va_{2t}^\dagger\right)|0\rangle, \quad u^2 + v^2 = 1. \qquad (1)$$

The lowest levels of the Landau quantization in the Landau gauge are characterized by the quantum number $n = 0$ and the uni-dimensional wave number t. $|0\rangle$ is the vacuum state. The equality $u^2 = v^2 = 1/2$ reflects the half-filling of the LLL in each layer. Introducing the hole operator $d_t^\dagger$, d_t for the first layer instead of the operators $a_{1t}^\dagger$ and a_{1t} the function (1) was transcribed in the form

$$|\psi\rangle = \prod_t \left(u + va_t^\dagger d_{-t}^\dagger\right)|\psi_0\rangle, \quad |\psi_0\rangle = \prod_t a_{1t}^\dagger |0\rangle$$
$$a_{2t} = a_t, \ a_{2t}^\dagger = a_t^\dagger, \ a_{1t} = d_{-t}^\dagger, \ a_{1t}^\dagger = d_{-t}. \qquad (2)$$

The operators $a_t^\dagger d_{-t}^\dagger$ create the electron-hole pairs with total wave vector equal to zero. The wave function (2) can be interpreted as describing the BEC of the QHExs [44–49].

The system we are interested in has only one layer, with electrons in conduction band and holes in the valence band of the same layer created by optical excitation or by p-n doping injection (both of these methods can be called "pumping"). In this case there is an intrinsic metastability, since electrons in the conduction band can drop down into the valence band and recombine with holes there. But we assume that the recombination rate of the electrons with holes has such a slow rate that the number of electrons and holes is nearly conserved. Unlike the case of the bilayer electron system with a half-filled lowest Landau level, in the case of a single excited layer which we consider, the density of excitons can be quite low, so that the electron Landau level and the separate hole Landau level are each only slightly occupied, and Pauli exclusion and phase space filling do not come in to play.

Our result concerning the BEC at $T = 0$ are estimations able to describe the real situation at finite temperatures lower than the critical temperature of the Berezinskii-Kosterlitz-Thouless (BKT) topological phase transition [50–52] related with the existence of the vortices and their clasters such as bound vortex-antivortex pairs. Just the unbinding of these pairs determines the critical temperature $T_{BKT} = \frac{\pi n\hbar^2}{2mk_B}$, where n is the surface density of the Bose particles and m is their mass. On one side of the phase transition there is a quasi-ordered fluid and on the other is a disordered unbounded vortex plasma. Although the formation of an isolated vortex will not occur at low temperature due its extensive creation energy, there always can be production of a pair of vortices with equal and opposite charges since the perturbation produced by such a pair falls off sufficiently rapidly at large distances so that their energy is finite [51,52]. Such topological formations can be easily created by the thermal fluctuations. The presence of the vortex clasters makes the earlier infinite homogeneous 2D e-h system to be nonhomogeneous as a whole. But the local homogeneity with finite local surface areas can exist leading to the BEC with finite critical temperature $T_c = \frac{2\pi n\hbar^2}{mk_B \lg(nS)}$ (Ref. [53]). Instead of an off diagonal long-range order as in the case of 3D Bose-gas in the 2D systems, there is only a long rang correlations which decays algebraically with distance. In such a way the quantum vortices promote the BEC and the formation of the superfluid component of the 2D Bose-gas at finite temperatures and at the same time the superfluid component is necessary for the formation of the quantum vortices. It is a self-organization-type situation. The BKT phase transition is a widely studied phenomenon [54–56].

We are interested in the distribution of the flux quanta in the case of an electron-hole system with equal average numbers of electrons and holes $\bar{N}_e = \bar{N}_h$ with filling factor $\nu = \bar{N}_e/N$, where N is the total number of flux quanta. In the case of fractional integer filling factor there is an integer number of flux quanta per each e-h pair. The creation of the vortices in this case is not studied at the present time.