Tuning Controllers in the Multiple-Loop Feedback Control System to the Objects with Inertia, Time Delay and Non Minimal Phase

Irina COJUHARI
Technical University of Moldova
Ştefan cel Mare av., 168, MD 2004, Chişinău, Republic of Moldova
cojuhari_irina@yahoo.com

Abstract **— This paper proposes a tuning algorithm of linear controllers P, PI, PID in the multiple-loop feedback control systems. The control object consists of two subprocesses, which are described by the dynamical models with inertia (first and second order), time delay and non minimal phase. The controllers in the internal contour and in the external contour tuning use the maximal stability degree method. P and PI controllers are used in the internal contour and P, PI, PID controllers are used in the external contour.**

Index Terms **— multiple-loop feedback control system, tuning of controllers, internal contour, external contour, non minimal phase, time delay, maximal stability degree method**

I. INTRODUCTION

Many tuning methods of typical controllers are used at the projecting of multiple-loop control systems: frequency method, criteria (of modulus) method etc [1,2,3,4]. The procedure of tuning controllers in the multiple-loop feedback control system becomes difficult. In this paper, it is proposed to use the maximal stability degree method (M.S.D) for tuning of typical controllers P, PI, PID for a class of control objects' models with inertia, which are connected in cascade, represented by two subprocesses and, as result with two regulating loops.

II. THE TUNING ALGORITHM OF CONTROLLERS

The multiple-loop feedback control system is represented by two contours: internal contour with controller's transfer function $H_{R2}(s)$ and subprocess $H_{F2}(s)$, and external contour with controller's transfer function $H_{R1}(s)$ and subprocess $H_{F1}(s)$ (Fig. 1). It is recommended to carry out the tuning of controllers first in the internal contour then in the external contour.

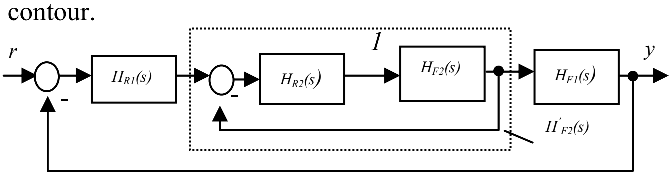

Figure 1. The multiple-loop feedback control system.

The control object consists of two inertial subprocesses with the transfer functions:

$$H_{F1}(s)=\frac{k_1(1-T_1s)}{(T_2s+1)(T_3s+1)}=\frac{b_1-b_0s}{a_0s^2+a_1s+a_2}, \quad (1)$$

where $b_0=k_1T_1; b_1=k_1; a_0=T_2T_3,\ a_1=T_2+T_3,\ a_2=1,$

$$H_{F2}(s)=\frac{k_2e^{-\tau s}}{T_4s+1}, \text{ with } T_4<T_3,\ T_2 \quad (2)$$

In expressions (1) and (2) we have the notations: k_1, k_2 are transfer coefficients of the subprocesses; T_1, T_2, T_3, T_4 are time constants of respective subprocesses, τ - time delay.

III. THE TUNING CONTROLLERS IN THE INTERNAL CONTOUR

The tuning of controller with transfer function (t.f.) $H_{R2}(s)$ from internal contour to the subprocess with t.f. $H_{F2}(s)$ is implemented. We assume that P and PI controllers are used.

P controller is tuned to the object with transfer function (2), applied M.S.D. method and tuning parameters of controller are determined from relation:

$$k_{p2}=\frac{e^{-\tau J}}{k_2}(T_4J-1) \quad (3)$$

In the relation (3) J is the maximal stability degree which is chosen from the following condition $J>0$.

To determinate the t.f. of the internal contour in case of tuning P and PI controllers it is proposed to approximate the value $e^{-\tau s}$ with Pade approximant [2]:

$$e^{-\tau s}=\frac{1}{\tau s+1} \quad (4)$$

The transfer function of the P controller is

$$H_{R2}(s)=k_{p2} \quad (5)$$

The t.f. of internal contour with P controller tuning is:

$$H'_{F2}(s)=\frac{H_{R2}(s)\cdot H_{F2}(s)}{1+H_{R2}(s)\cdot H_{F2}(s)}=\frac{k'}{l_0s^2+l_1s+l_2}, \quad (6)$$

where $k'=\frac{k_{p2}k_2}{1+k_{p2}k_2}; l_0=\frac{\tau T_4}{1+k_{p2}k_2};\ l_1=\frac{\tau+T_4}{1+k_{p2}k_2};\ l_2=1.$

PI controller is tuned to the object with the transfer function (2), applied the M.S.D. method and tuning parameters of controller are determined from relations

$$k_{p2}=\frac{e^{-\tau J}}{k_2}(-\tau T_4J^2+(\tau+2T_4)J-1) \quad (7)$$

$$k_{i2}=\frac{e^{-\tau J}}{k_2}J^2(-\tau T_4J+\tau+T_4) \quad (8)$$

We can obtain the values of parameters k_{p2}, k_{i2}, changing the $J>0$ value, for that the performances of control system are sated.

The transfer function of the PI controller is

$$H_{R2}(s) = k_{p2} + \frac{k_{i2}}{s} \tag{9}$$

The t.f. of internal contour with PI controller tuning, using expression (4) is:

$$H'_{F2}(s) = \frac{H_{R2}(s)H_{F2}(s)}{1 + H_{R2}(s)H_{F2}(s)} = \frac{l_0 s + l_1}{c_0 s^3 + c_1 s^2 + c_2 s + c_3} \tag{10}$$

where $l_0 = \frac{k_{p2}}{k_{i2}};\ l_1 = 1;\ c_0 = \frac{\tau T_4}{k_{i2}k_2};\ c_1 = \frac{(\tau + T_4)}{k_{i2}k_2};$

$$c_2 = \frac{1 + k_{p2}k_2}{k_{i2}k_2}; c_3 = 1.$$

IV. THE TUNING CONTROLLERS IN THE EXTERNAL CONTOUR

The structure block scheme of external contour is represented in the Fig. 2 *a, b*.

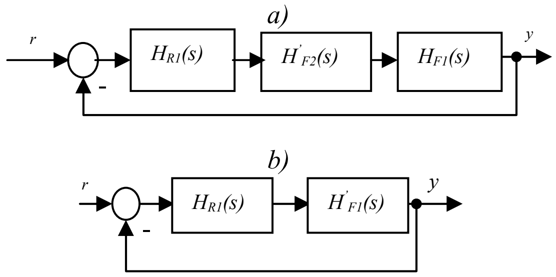

Figure 2. Structure block schema of external contour.

For the tuning of P, PI, PID controllers in the external contour it is necessary to determine the equivalent transfer function of object (6) with P controller tuning in the internal contour with the t.f. of subprocess (1)

$$H'_{F1}(s) = H'_{F2}(s)H_{F1}(s) = \frac{k'}{(l_0 s^2 + l_1 s + l_2)} \cdot \frac{k_1(1 - T_1 s)}{(T_2 s + 1)(T_3 s + 1)} = \tag{11}$$

$$= \frac{b_1 - b_0 s}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4},$$

where $b_0 = \frac{k_{p2}k_2k_1T_1}{1 + k_{p2}k_2}; b_1 = \frac{k_{p2}k_2k_1}{1 + k_{p2}k_2}; a_0 = \frac{\tau T_2 T_3 T_4}{1 + k_{p2}k_2};$

$$a_1 = \frac{T_2T_3(\tau + T_4)}{1 + k_{p2}k_2} + \frac{\tau T_4(T_2 + T_3)}{1 + k_{p2}k_2};$$

$$a_2 = T_2T_3 + \frac{(T_2 + T_3)(\tau + T_4)}{1 + k_{p2}k_2} + \frac{\tau T_4}{1 + k_{p2}k_2};$$

$$a_3 = (T_2 + T_3) + \frac{\tau + T_4}{1 + k_{p2}k_2}; a_4 = 1.$$

For object with t.f. (11) P, PI, PID controllers can be tuned by applying the M.S.D. method using the relation from [5, 6].

Control system with P controller:

$$k_{p1} = \frac{-a_0 J^4 + a_1 J^3 - a_2 J^2 + a_3 J - a_4}{b_0 J + b_1} \tag{12}$$

Control system with PI controller:

$$k_{p1} = \frac{-d_0 J^5 + d_1 J^4 - d_2 J^3 + d_3 J^2 + d_4 J - d_5}{(b_0 J + b_1)^2} \tag{13}$$

where $d_0 = 4a_0 b_0;\ d_1 = 3a_1 b_0 - 5a_0 b_1;$

$d_2 = 2a_2 b_0 - 4a_1 b_1;$

$d_3 = a_3 b_0 - 3a_2 b_1;$

$d_4 = 2a_3 b_1;\ d_5 = a_4 b_1.$

$$k_{i1} = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2 + a_4 J}{b_0 J + b_1} + k_{p1} J \tag{14}$$

Control system with PID controller:

$$k_{d1} = \frac{d_0 J^6 - d_1 J^5 + d_2 J^4 - d_3 J^3 + d_4 J^2 - d_5 J + d_6}{2(b_0 J + b_1)^4}, \tag{15}$$

where $d_0 = 3d_0 b_0^2; d_1 = 2d_1 b_0^2 - 8d_0 b_0 b_1;$

$d_0 = -3d_0 b_0^2; d_1 = 2d_1 b_0^2 - 8d_0 b_0 b_1;$

$d_2 = 5d_0 b_1^2 - 6d_1 b_0 b_1 + d_2 b_0^2;$

$d_3 = 4d_1 b_1^2 - 4d_2 b_0 b_1;$

$d_4 = 3d_2 b_1^2 - 2d_3 b_0 b_1 + d_4 b_0^2;$

$d_5 = 2d_3 b_1^2 + 2d_5 b_0^2;$

$d_6 = -5d_4 b_1^2 - 2d_5 b_0 b_1,$

$$k_{p1} = \frac{-d_0 J^5 + d_1 J^4 - d_2 J^3 + d_3 J^2 + d_4 J - d_5}{(b_0 J + b_1)^2} + 2k_{d1} J \tag{16}$$

where $d_0 = 4a_0 b_0;\ d_1 = 3a_1 b_0 - 5a_0 b_1;$

$d_2 = 2a_2 b_0 - 4a_1 b_1; d_3 = a_3 b_0 - 3a_2 b_1;\ d_4 = 2a_3 b_1; d_5 = a_4 b_1,$

$$k_{i1} = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2 + a_4 J}{b_0 J + b_1} - k_{d1} J^2 + k_{p1} J. \tag{17}$$

For the tuning of P, PI, PID controllers in the external contour it is necessary to determine the equivalent t.f of object (10) with PI controller in the internal contour with the t.f. of subprocess (1)

$$H''_{F1}(s) = H'_{F2}(s)H_{F1}(s) = \frac{l_0 s + l_1}{c_0 s^3 + c_1 s^2 + c_2 s + c_3} \cdot \frac{k_1(1 - T_1 s)}{(T_2 s + 1)(T_3 s + 1)} = \tag{18}$$

$$= \frac{-b_0 s^2 + b_1 s + b_2}{a_0 s^5 + a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s + a_5},$$

where

$$b_0 = \frac{k_{p2}k_1T_1}{k_{i2}};\quad b_1 = \frac{k_{p2}k_1}{k_{i2}} - k_1T_1;\quad b_2 = k_1;$$

$$a_0 = \frac{\tau T_2 T_3 T_4}{k_{i2}k_2};\quad a_1 = \frac{(\tau + T_4)T_2T_3}{k_{i2}k_2} + \frac{\tau T_4 (T_2 + T_3)}{k_{i2}k_2};$$

$$a_2 = \frac{(1 + k_{p2}k_2)T_2T_3}{k_{i2}k_2} + \frac{(\tau + T_4)(T_2 + T_3)}{k_{i2}k_2} + \frac{\tau T_4}{k_{i2}k_2};$$

$$a_3 = T_2T_3 + \frac{(1 + k_{p2}k_2)(T_2 + T_3)}{k_{i2}k_2} + \frac{(\tau + T_4)}{k_{i2}k_2};$$

$$a_4 = T_2 + T_3 + \frac{1 + k_{p2}k_2}{k_{i2}k_2};\ a_5 = 1.$$

For object with t.f. (18) P, PI, PID controllers can be tuned by applying the M.S.D. method using the relations from [5, 6].

Control system with P controller

$$k_{p1} = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2 + a_4 J - a_5}{-b_0 J^2 - b_1 J + b_2} \tag{19}$$

Control system with PI controller

$$k_{p1}=\frac{-d_0J^7+d_1J^6-d_2J^5+d_3J^4-d_4J^3+d_5J^2+d_6J-d_7}{\left(-b_0J^2-b_1J+b_2\right)^2}; \quad (20)$$

where

$$d_0=4a_0b_0;\quad d_1=3a_1b_0-5a_0b_1;$$
$$d_2=2a_2b_0-4a_1b_1-6a_0b_2;$$
$$d_3=a_3b_0-3a_2b_1-5a_1b_2; \quad (21)$$
$$d_4=-2a_3b_1-4a_2b_2;$$
$$d_5=-a_5b_0-a_4b_1-3a_3b_2;$$
$$d_6=2a_4b_2;$$
$$d_7=a_5b_2;$$

$$k_{i1}=\frac{-a_0J^6+a_1J^5-a_2J^4+a_3J^3-a_4J^2+a_5J}{-b_0J^2-b_1J+b_2}+k_{p1}J. \quad (22)$$

Control system with PID controller

$$k_{d1}=\frac{l_0J^{10}-l_1J^9-l_2J^8-l_3J^7-l_4J^6-l_5J^5-l_6J^4-l_7J^3-l_8J^2-l_9J-l_{10}}{2(-b_0J^2-b_1J+b_2)^4}, \quad (23)$$

where $l_0=-3d_0b_0^{\ 2}; l_1=-8d_0b_0\ b_1+2d_1b_0^{\ 2};$

$$l_2=10d_0b_0b_2+6d_1b_0b_1-d_2b_0^{\ 2}-5d_0b_1^2;$$
$$l_3=-4d_2b_0b_1+12d_0b_1b_2-8d_1b_0b_2+4d_1b_1^{\ 2};$$
$$l_4=6d_2b_0b_2-3d_2b_1^2+d_4b_0^2-7d_0b_2^{\ 2}-10d_1b_1b_2+2d_3b_0b_1;$$
$$l_5=8d_2b_1b_2-4d_3b_0b_2-2d_5b_0^2+6d_1b_2^2+2d_3b_1^2;$$
$$l_6=-d_4b_1^2-6d_3b_1b_2+2d_4b_0b_2-3d_6b_0^2-2d_5b_0b_1-5d_2b_2^2;$$
$$l_7=4d_7b_0^2+4d_3b_2^2+4d_4b_1b_2-4d_6b_0b_1;$$
$$l_8=-d_6b_1^2-2d_5b_1b_2+2d_6b_0b_2-3d_4b_2^2+6d_7b_0b_1;$$
$$l_9=2d_7b_1^2-4d_7b_0b_2+2d_5b_2^2;$$
$$l_{10}=-2d_7b_1b_2+d_6b_2^2,$$

where $d_0,d_1,d_2,d_3,d_4,d_5,d_6,d_7$ are determined from relations (21),

$$k_{p1}=\frac{-d_0J^7+d_1J^6-d_2J^5+d_3J^4-d_4J^3+d_5J^2+d_6J-d_7}{\left(-b_0J^2-b_1J+b_2\right)^2}+2k_dJ; \quad (24)$$

where $d_0=4a_0b_0;\quad d_1=3a_1b_0-5a_0b_1;$

$$d_2=2a_2b_0-4a_1b_1-6a_0b_2;$$
$$d_3=a_3b_0-3a_2b_1-5a_1b_2;$$
$$d_4=-2a_3b_1-4a_2b_2;$$
$$d_5=-a_5b_0-a_4b_1-3a_3b_2;$$
$$d_6=2a_4b_2;\quad d_7=a_5b_2;$$
$$k_{i1}=\frac{-a_0J^6+a_1J^5-a_2J^4+a_3J^3-a_4J^2+a_5J}{-b_0J^2-b_1J+b_2}-k_{d1}J^2+k_{p1}J. \quad (25)$$

Values of k_{p1},k_{i1},k_{d1} parameters are obtained, varied the J value, for that the performances of control system are sated. The procedure of determining the optimal coefficients k_p,k_i,k_d from expressions (3), (7), (8), (12)-(17), (19)-(20), (22)-(25) for that the control system will have the sated performance, is a difficult one.

The following procedure is proposed to determine the optimal values of parameters k_p,k_i,k_d from relations (3), (7), (8), (12)-(17), (19)-(20), (22)-(25) which represent the dependences of the maximal stability degree *J*. The variable *J* is changing and the curves $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ for the respectively controller and object were obtained. After, the sets of values of the k_p,k_i,k_d parameters get the optimal and quasioptimal value of *J*. For each set of values of the k_p,k_i,k_d parameters it was making the simulation and it was determined the transition process for that the obtained performance corresponding the sated performance.

V. APLICATIONS AND COMPUTER SIMULATION

To show the efficiency of the proposed algorithm of tuning the typical controllers in the multiple-loop feedback control system with inertia (third order) using the presented relations an example with the object model which has the following parameters: $H_{F2}(s)$- $k_2=2$, $T_4=3$, $\tau=1$ and $H_{F1}(s)$ – $k_1=2$, $T_1=2$, $T_2=10$, $T_3=5$ was examined.

The P, PI controllers were tuning in internal contour using the maximal stability degree method, which permitted to obtain the high performance, varied values of *J* and choosing the k_{p2} and k_{p2}, k_{i2} values for the respective controllers.

The P, PI and PID controllers were tuned in external contour using the maximal stability degree method, which permitted to obtain the high performance, varied values of *J* and choosing the respectively values of the P, PI, PID controllers.

The computer simulation has been made in MATLAB and the simulation diagram of multiple-loop feedback control system is presented in the Fig. 3.

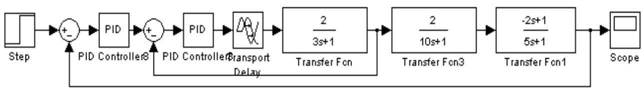

Figure 3. Simulation diagrams of the control system.

The transition processes of the multiple-loop feedback control system for external contour is presented in the Fig. 4: *a)* – transition process in the external contour, with P controller tuning in the internal contour (*k_{p2}=0.395*): external contour with P controller (*k_{p1}=0.0765*) - curve *1*; PI controller with optimal parameters *(J_{opt}=0.09, k_{p1}=0.394, k_{i1}= 0.0393)* – curve *2*; PID controller with optimal values *(J_{opt}=0.2, k_{p1opt}=1.715, k_{i1opt}= 0.121, k_{dopt}=5.544)* – curve *3;* optimization in Matlab *(k_{p1}=2.0463, k_{i1}= 0.1745, k_d=5.53356)* – curve *4;* with PID controller with values *(J=0.1, k_{p1}=0.678, k_{i1}= 0.052, k_{d1}=1.52)* – curve *5;* PID controller with values *(J=0.3, k_{p1}=0.87, k_{i1}= 0.01, k_{d1}= 3.922)* – curve *6; b)* – transition process in the external contour with PI controller tuning in the internal contour (*J_{opt}=0.74, k_{p2}=0.605, k_{i2}= 0.232*): external contour with P controller (*k_{p1}=0.0361*) - curve *1*; with PI controller with optimal values *(J_{opt}=0.09, k_{p1}=0.177, k_{i1}= 0.0176)* - curve *2*; PID controller with values *(J_{opt}=0.23, k_{p1}=1.736, k_{i1}= 0.128, k_{d1}=5.461)* - curve *3;* optimization in Matlab *(k_{p1}=1.7369, k_{i1}= 0.1153, k_d=5.461)* – curve *4;* PID controller with values *(J=0.1, k_{p1}=0.539, k_{i1}= 0.042, k_{d1}=1.11)* – curve *5;* PID controller with values *(J=0.38, k_{p1}=1.196, k_{i1}= 0.043, k_{d1}=4.594)* – curve *6.*

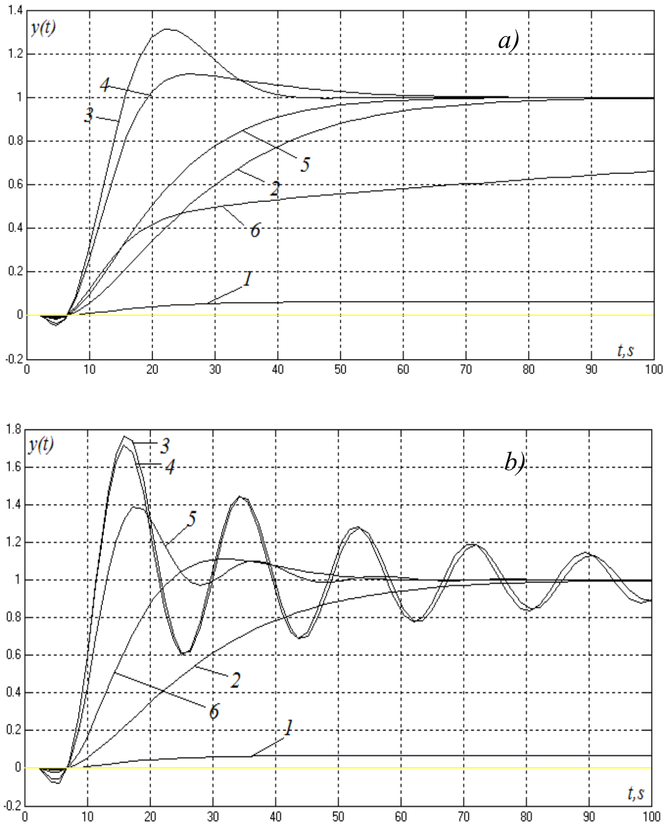

Figure 4. Transition processes of the multiple-loop feedback control system.

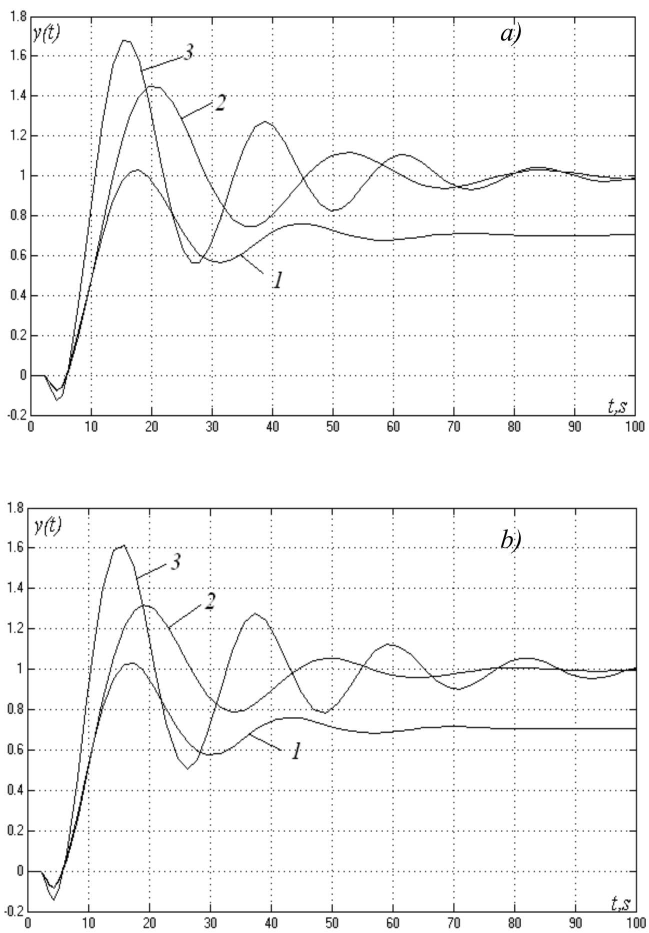

Figure 5. Transition processes of the multiple-loop feedback control system.

To compare the obtained results the tuning of the typical controller using Ziegler-Nichols method was made. The Fig. 5 presents the transition processes of the multiple-loop feedback control system for external contour in the case of using Ziegler-Nichols method: *a)* – transition process in the external contour, with P controller in the internal contour ($k_{p2}=1.35$): external contour with P controller ($k_{p1}=1.65$) - curve *1*; with PI controller with optimal parameters *(* $k_{p1}=1.485$, $k_{i1}= 0.078$*)* – curve *2*; with PID controller with optimal parameters *(*$k_{p1}=2.475$, $k_{i1}= 0.1042$, $k_d=1.6$*)* – curve *3; b)* – transition process in the external contour with PI controller in the internal contour ($k_{p2}=1.215$, $k_{i2}= 0.357$): external contour with P controller ($k_{p1}=1.175$) - curve *1*; with PI controller with optimal parameters *(*$k_{p1}=1.057$, $k_{i1}= 0.073$*)* - curve *2*; with PID controller with parameters *(*$k_{p1}=1.762$, $k_{i1}= 0.098$, $k_{d1}=1.7$ *)* – curve *3*.

The Fig. 6 *a)* presents the transition processes of the multiple-loop feedback control system, the Fig. 6 *b)* presents the repartition of the poles for the following cases: *1*- control system with P controller tuning in the internal contour and PID in the external contour using the maximal stability degree method, *2* - control system with PI controller tuning in the internal contour and PID in the external contour using the maximal stability degree method, *3*- control system with P controller tuning in the internal contour and PID in the external contour using the Ziegler – Nichols method, *4*- control system with PI controller tuning in the internal contour and PID in the external contour using the Ziegler – Nichols method.

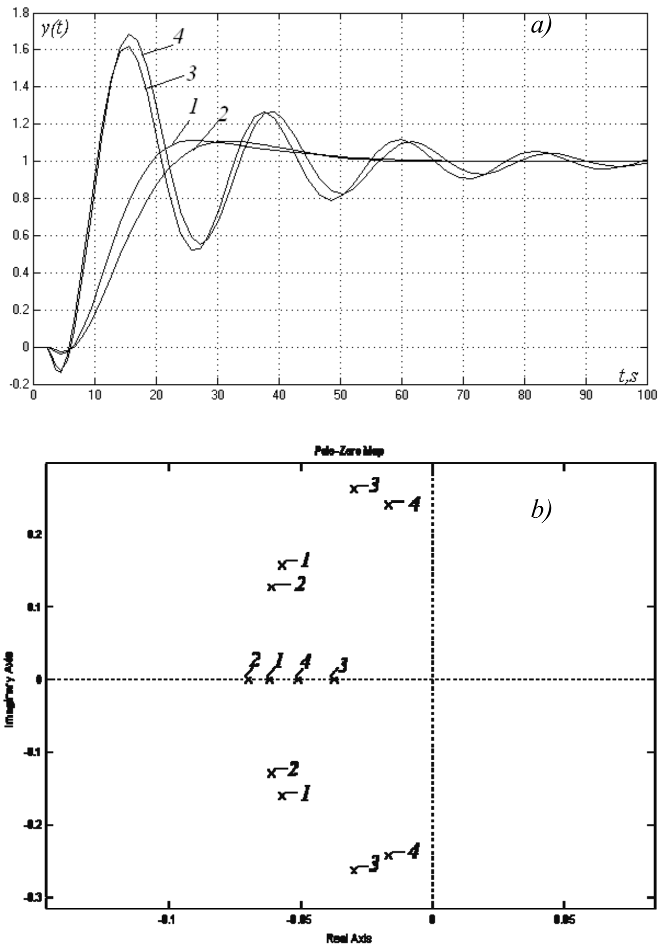

Figure 6. Transition processes of the multiple-loop feedback control system (a) and presentation of distribution of the systems' poles (b).

VI. CONCLUSIONS

As a result, after tuning the P, PI, PID controller to the multiple-loop feedback control system with object's models (1), (2) with known parameters, the following conclusions can be made:

1. The tuning of P, PI controller in the internal contour in conformity with the maximal stability degree method permitted to obtain the high results varying the value of the $J>0$ and choosing the parameters of the respectively controllers for obtaining the sated performance of the internal contour.
2. The tuning of P, PI, PID controllers in the external contour using the maximal stability degree method permitted to obtain the high results varying the value of the $J>0$ and obtained the optimal value and the suboptimal values of the J, and choosing the set of the values of controllers' parameters for obtaining the sated performances.

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