# ESTIMATION OF THE MATHEMATICAL MODEL OF THE DC MOTOR COUPLED WITH A REACTION WHEEL

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Abstract

In this paper was proposed to estimate the mathematical model of the DC motor coupled with a reaction wheel. It was proposed the experimental curve identification and its approximation with model of object with first, second and third order inertia. For the obtained transfer function was proposed to tune P, PI and PID controllers based on the maximum stability degree method with iterations.

**Keywords:** estimation of the mathematical model, experimental identification, transfer function, control algorithm, control system, maximum stability degree with iterations. **2010 MSC:** 93B30, 93B40.

#### 1. INTRODUCTION

Industrial processes can be classified into technological processes and information processes. Technological process means a succession of operations on raw materials, semi-finished products, by-products, in order to obtain high-performance products. Operations are performed on technology facilities and involve mass and energy transfer. An important benefit for technological installations is given by the development of electrical machines, which represent actuators and in most cases operate on the basis of the electromagnetic forces acting on a magnetic field current conductor. Electric motors can be classified by the type of electric current: DC motors and AC motors. Due to the linearity of its features and relatively simple speed control methods, DC motors are the most used motors as actuators in the control systems.

The DC motor is frequently encountered in automatic control systems either as an actuator, or as a control object, where in both situations it is necessary to control its speed, which is solved by using a typical control algorithm [9].

In order to solve the control problem, it is often necessary to be known the mathematical model of the described industrial process. Knowing the mathematical model of the industrial process requires the use of identification

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procedures. The identification of the industrial processes is the estimation of the parameters and the structure of the mathematical model, ensuring the best coincidence of the output signal from the model with the output of the process, where the physical process is regarded as an entity, seen as a black box about the internal structure of which no details are known. There are two categories of techniques for identification the mathematical model of the physical process undergoing identification:

- 1. Analytical identification. In this case the identification model is obtained on the basis of the physico-chemical laws, which generate the process dynamics.
- 2. Experimental identification. In this case, the mathematical model is otained based on the process input and output variables associated with the process [1]-[3].

In the present paper it is proposed to perform the experimental identification of the mathematical model of the DC motor coupled with a reaction wheel. The purpose of this research is to choose the reaction wheels and determine the control algorithm for stabilization the microsatellite witch is developed at the National Centre of Space Technologies. In order to choose the reaction wheels and analyze the satellite stabilization process, it is necessary to perform the mathematical modeling and computer simulation of the satellite stabilization.

#### 2. DESCRIPTION OF THE DESIGNED SYSTEM

The FK130SH DC motor has been chosen as an actuator for testing several reaction wheels. The system was implemented based on the NUCLEO-F303K8 platform from ST Microelectronics. The reaction wheel is coupled directly to the motor. The block diagram of the motor speed control with reaction wheel is shown in figure 1. The speed of the motor coupled with the reaction wheel is controlled by the STM32F303K8 microcontroller.

To start recording the data, the computer sends a start message to the microcontroller, which controls the electronic key on the motor. The instantaneous rotation speed of the motor is measured by the microcontroller and transmitted to the computer in real time. As the speed sensor, the EE-SX4235A-P2 transmissive photomicrosensor is used, which transmits logical "one", when between the photoemitter and the photoreceptor is an opaque object, in the case of the reaction wheel marker, thus generating a pulse at each marker crossing through the sensor aperture.

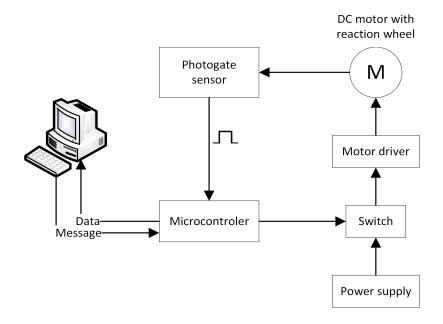


Fig. 1. Structure diagram.

## 3. IDENTIFICATION OF THE CONTROL OBJECT

The experimental identification involves the acquisition of data, so that the experimental variation of the DC motor speeds at the reference speed of 8500 rpm was obtained as presented in the figure 2.

To estimate the mathematical model of the control object it was proposed to use the module Process Models from System Identification Toolbox from MATLAB.

It was proposed to approximate the control object with three types of the mathematical models:

1. Model of object with first order inertia

$$H_1(s) = \frac{k}{Ts+1} = \frac{0.98006}{2.9173s+1}. (1)$$

2. Model of object with second order inertia

$$H_2(s) = \frac{k}{(T_1s+1)(T_2s+1)} = \frac{1.0069}{3.1695s^2 + 5.02899s + 1}.$$
 (2)

3. Model of object with third order inertia

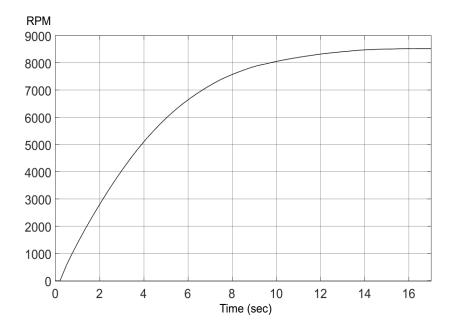


Fig. 2. Experimental curve.

$$H_3(s) = \frac{k}{(T_1s+1)(T_2s+1)(T_3s+1)} = \frac{4.2292}{19044.8s^3 + 37394.9s^2 + 5713.25s + 1}.$$
 (3)

In the transfer functions (1), (2) and (3) are used the following notations: k is the transfer coefficient;  $T_1$ ,  $T_2$ ,  $T_3$  - time constants.

The comparison of the results is shown in figure 3, where 1 - curve is transient precess obtained for (1) identified model, 2 - curve is transient precess obtained for (2) identified model and 3 - curve is transient precess obtained for (3) identified model.

### 4. SYNTHESIS OF THE TYPICAL CONTROLLERS

The structure scheme of the automatic control system is presented in the figure 4, where  $H_R(s)$  represents the controller transfer function and  $H_{PF}(s)$  the transfer function of the control object presented in the relation (2).

It was proposed to tune P, PI and PID controller based on the maximum stability degree method with iterations. It was proposed to synthesize the

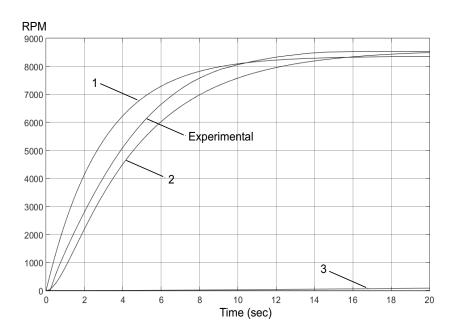
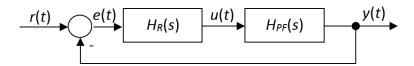


Fig. 3. Transient processes.



 ${\it Fig.}\ \ \, {\it 4.}\ \ \, {\it Structure scheme of the control system}.$ 

standard control algorithms P, PI and PID to the identified model of object (2), which are described by the following transfer functions:

$$H_P(s) = k_p. (4)$$

$$H_{PI}(s) = k_p + \frac{k_i}{s}. (5)$$

$$H_{PID}(s) = k_p + \frac{k_i}{s} + k_d s. \tag{6}$$

where  $k_p$ ,  $k_i$ ,  $k_d$  represent the tuning parameters of the respectively controller [4].

As tuning methods, it was proposed to use maximum stability degree method with iterations [5]-[8].

According to the maximum stability degree method with iterations, there were obtained the dependencies  $k_p = f(J)$  for P controller,  $k_p = f(J)$ ,  $k_i = f(J)$  for PI controller and  $k_p = f(J)$ ,  $k_i = f(J)$ ,  $k_d = f(J)$  for PID controller:

- For P controller

$$k_p = \frac{1}{k}(-a_0J^2 + a_1J - a_2). (7)$$

- For PI controller

$$k_p = \frac{1}{k}(-3a_0J^2 + 2a_1J - a_2).$$

$$k_i = \frac{J^2}{k}(-2a_0J + a_1).$$
(8)

- For PID controller

$$k_d = \frac{1}{k}(3a_0J - a_1).$$

$$k_p = \frac{1}{k}(3a_0J^2 - a_2).$$

$$k_i = \frac{a_0}{k}J^3.$$
(9)

According to the obtained relationships (7)-(9) were obtained the dependencies  $k_p = f(J)$  (for P controller, figure 5),  $k_p = f(J)$ ,  $k_i = f(J)$  (for the PI controller, figure 6) and  $k_p = f(J)$ ,  $k_i = f(J)$ ,  $k_d = f(J)$  (for the PID controller, figure 7), where J is the maximum stability degree.

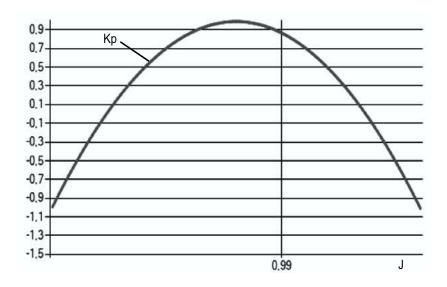


Fig. 5. Dependence  $k_p = f(J)$ , for P controller.

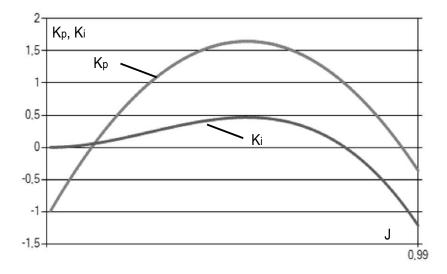


Fig. 6. Dependences  $k_p = f(J)$ ,  $k_i = f(J)$ , for PI controller.

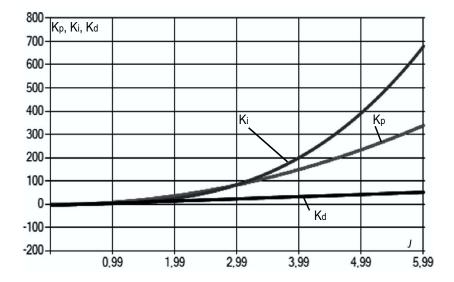


Fig. 7. Dependences  $k_p = f(J)$ ,  $k_i = f(J)$ ,  $k_d = f(J)$ , for PID controller.

From curves presented in the figures 5, 6, 7 were chosen the sets of tuning values for the case of tuning P, PI and PID controllers. The obtained sets of values  $J - k_p, k_i, k_d$  are presented in the table 1.

Table 1. The tunung parameters of P, PI, PID controllers.

Type of controller	No.	J	$k_p$	$k_i$	$k_d$
P controller	1	0,79	0,98		
PI controller	2	0.53	1,64	0,46	
PID controller	3	1.09	10.226	4.07	5.298

Based on the obtained results it was done the computer simulation of the control system, using the MATLAB Simulink and the obtained transient processes are presented in the figure 8. The numbering of curves corresponds to the numbering from the table 1.

### 5. CONCLUSIONS

Stabilization of the Microsatellite from the National Centre of Space Technologies is also carried out under the control of the reaction wheels and magnetotorqers. In order to choose the reaction wheels which are important in the satellite stabilization process, was proposed to design the control system of the DC motor speed and was identified the mathematical model of the speed

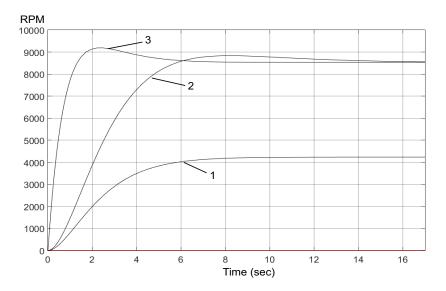


Fig. 8. Transient processes.

variation of the DC motor. As the results, it was obtained three types of model objects (1)-(3) with inertia first, second and third order. The satisfied result of identification was obtained for the case of approximation the experimental curve with transfer function with second order inertia, and to this model of object was proposed to tune the P, PI and PID controller by the maximum stability degree method with iterations. Analyzing the obtained results, it was observed that MATLAB offers different methods for identification, however the obtained results demonstrate that the estimated models have error around 10 percent and as the next research it is proposed to develop methods with less error of approximation.

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