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CONTRIBUTIONS OF THE CONTROLLERS TUNING IN THE MULTIPLE-LOOP FEEDBACK CONTROL SYSTEM WITH THREE CONTOURS WITH INERTIA

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Abstract **– - A tuning algorithm of linear controllers P, PI, PID in multiple-loop feedback control systems is proposed in this paper. The control objects consist of three subprocesses, which are described by dynamical models with inertia (third order). The controllers in the internal contours 1 and 2 and in the external contour are tuning according the maximal stability degree method. In the internal contour is used controllers P and PI, in the external contour is used controllers P, PI, PID. There are using the iterative procedure, for determinate the optimal parameters of controllers P, PI, PID. The tuning algorithm of controllers represents an algebraic method, which consists of two stages. On first stage the numerical value of the optimum stability degree of designed control system is determined. On the second stage the numerical value of tuning dynamic parameters of controllers are determined from algebraic expressions.**

Keywords: *multiple-loop feedback control system, tuning of controllers, maximal stability degree method.*

1. INTRODUCTION

At the projecting of multiple-loop control systems are used many tuning methods of typical controllers: frequency method, criteria (of modulus) method etc. Frequency method is accompanied with difficulties of calculating [1,2,3]. Criteria (of modulus) method becomes unacceptable when the control processes are slowly, and they have big time constants and this reduce the performances of entire system [1,2,3]. To bypass these above-cited inconveniences in the paper is proposed to use of the maximal stability degree method for tuning of typical controllers *P, PI, PID* for a class of control objects' models with inertia, which are connected in cascade, represented by three subprocesses and, as result with three regulating loops.

2. THE ALGORITHM OF TUNING CONTROLLERS

The multiple-loop feedback control system (Fig. 1) is represented by three contours: internal contour *1* with controller's transfer function $H_{R3}(s)$ and subprocess $H_{F3}(s)$, internal contour *2* with controller's transfer function $H_{R2}(s)$ and equivalent subprocess $H'_{F2}(s)$, and external contour with
controller's transfer function $H_{R1}(s)$ and equivalent subprocess $H'_{F1}(s)$.
The tuning of controllers is recommended to realize first in the inertial contour *1*, after in the second inertial contour, then in the external contour.
The control object consists from three inertial subprocesses with the transfer functions:

$$H_{F1}(s)=\frac{k_1}{T_1 s+1}, \quad (1)$$

$$H_{F2}(s)=\frac{k_2}{T_2 s+1}, \text{ with } T_1>T_2. \quad (2)$$

$$H_{F3}(s)=\frac{k_3}{T_3 s+1}, \text{ cu } T_1>T_2>T_3. \quad (3)$$

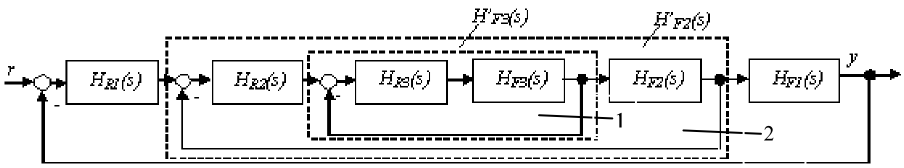

Fig.1 - The multiple-loop feedback control system

In expressions (1), (2) and (3) we have the notations: k_1, k_2, k_3 are transfer coefficients of subprocesses; T_1, T_2, T_3 are time constants of respective subprocesses.

2.1 The tuning controllers in the first internal contour

Is implementing the tuning of controller with transfer function (t.f.) $H_{R3}(s)$ from internal contour to the subproccess with t.f. $H_{F3}(s)$. Suppose that *P* and *PI* controllers are used.

- ***P* controller** is tuning to the object with transfer function (3), applied the maximal stability degree (M.S.D.) method and tuning parameters of controller are determined from relations [4,5,6]

$$k_{p3} = \frac{1}{k_3}(T_3 J - 1)\cdot \tag{4}$$

In the relation (4) *J* is the maximal stability degree and which is chose from the following condition *J>0*. The maximal stability degree *J* is choosing from the consideration as the duration of the transition process will has the sated values.
The t.f. of internal contour with *P* controller:

$$H'_{F3}(s) = \frac{H_{R3}(s)H_{F3}(s)}{1+H_{R3}(s)H_{F3}(s)} = \frac{k_3 k_{p3}}{T_3 s + 1 + k_3 k_{p3}} = \frac{k_3'}{T_3' s + 1}, \tag{5}$$

where $k_3' = k_3 k_{p3} / (1 + k_3 k_{p3}),\ T_3' = T_3 \ / (1 + k_3 k_{p3})$.

- ***PI* controller** is tuning to the object with the transfer function (3), applied the M.S.D. method and tuning parameters of controller are determined from relations [4,5,6]

$$k_{p3} = \frac{1}{k_3}(2T_3 J - 1), \tag{6}$$

$$k_{i3} = \frac{T_3}{k_3} J^2 . \tag{7}$$

We can obtain the values of parameters k_{p3}, k_{i3}, changing the *J>0* value, for that the performances of control system are predefined.
The t.f. of internal contour with *PI* controller is

$$H'_{F2}(s) = \frac{H_{R3}(s)H_{F3}(s)}{1+H_{R3}(s)H_{F3}(s)} = \frac{k_3 k_{p3} s + k_3 k_{i3}}{T_3 s^2 + s + k_3 k_{p3} s + k_3 k_{i3}} = \\ = \frac{d_0 s + d_1}{c_0 s^2 + c_1 s + c_2}, \tag{8}$$

where $d_0 = \frac{k_{p3}}{k_{i3}},\ d_1 = 1,\ c_0 = \frac{T_3}{k_3 k_{i3}},\ c_1 = \frac{1 + k_3 k_{p3}}{k_3 k_{i3}},\ c_2 = 1.$

2.2 The tuning controllers in the second internal contour

The structure block scheme of the second internal contour is represented in the figure 2.

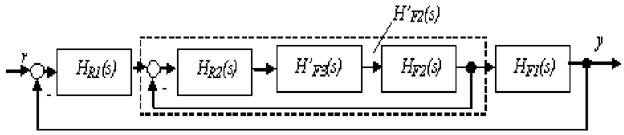

Fig.2 - Structure block schema of automation system with second internal contour

Is implementing the tuning of controller with t.f. $H_{R2}(s)$ from the second internal contour to the subproccess with t.f. $H'_{F3}(s)$ and $H_{F2}(s)$. Suppose that *P* and *PI* controllers are used, for tuning of *P, PI* controllers in the second internal contour it is necessary to determine the equivalent transfer function of the object (5) with *P* controller in the first internal contour and transfer function of subprocess $H_{F2}(s)$ (2)

$$H_{\Sigma F2}(s) = H'_{F3}(s) H_{F2}(s) = \frac{k_3'}{(T_3' s + 1)} \frac{k_2}{(T_2 s + 1)} = \frac{k_2'}{a_0 s^2 + a_1 s + a_2}, \tag{9}$$

where $a_0 = T_2 T_3',\ a_1 = T_2 + T_3',\ a_2 = 1,\ k_2' = k_2 k_3' = \frac{k_2 k_3 k_{p3}}{1 + k_3 k_{p3}}$.

- ***P* controller** is tuning to the object with transfer function (9), applied the M.S.D. method and tuning parameters of controller are determined from relations [5,6]

$$k_{p2} = \frac{1}{k_2'}(-a_0 J^2 + a_1 J - a_2) = \frac{1}{k_2'}\left(\frac{a_1^2}{4a_0} - 1\right) \text{ cu } J = \frac{a_1}{2a_0} . \tag{10}$$

In the relation (9) *J* is the maximal stability degree and which is chose from the following condition *J>0*.
The t.f. of the second internal contour with *P* controller:

$$H'_{F2}(s) = \frac{H_{R2}(s)H'_{F2}(s)}{1 + H_{R2}(s)H'_{F2}(s)} = \frac{k_2''}{a_0' s^2 + a_1' s + a_2'}, \tag{11}$$

where $a_0' = \frac{a_0}{a_2 + k_{p2} k_2'},\ a_1' = \frac{a_1}{a_2 + k_{p2} k_2'},\ a_2' = 1,\ k_2'' = k_{p2} k_2'.$

- ***PI* controller** is tuning to the object with the transfer function (8), applied the M.S.D. method and tuning parameters of controller are determined from relations [5,6]

$$k_{p2} = \frac{1}{k_2'}(-3a_0 J^2 + 2a_1 J - a_2) = \frac{1}{k_2'}\left(\frac{a_1^2}{3a_0} - 1\right), \tag{12}$$

$$k_{i2} = \frac{1}{k_2'}(a_0 J^3 - a_1 J^2 + a_2 J) + k_p J = \frac{1}{k_2'} \frac{a_1^3}{27 a_0^2} \text{ with } J = \frac{a_1}{3a_0} . \tag{13}$$

We can obtain the values of parameters k_{p2}, k_{i2}, changing the *J>0* value, for that the performances of control system are predefined.
The t.f. of internal contour with *PI* controller is

$$H'_{F2}(s) = \frac{H_{R2}(s)H_{F2}(s)}{1 + H_{R3}(s)H_{F3}(s)} = \frac{d_0 s + d_1}{c_0 s^3 + c_1 s^2 + c_2 s + c_3} . \tag{14}$$

where $d_0 = \frac{k_{p2}}{k_{i2}},\ d_1 = 1,\ c_0 = \frac{a_0}{k_2' k_{i2}},\ c_1 = \frac{a_1}{k_2' k_{i2}},$

$$c_2 = \frac{(a_2 + k_2' k_{p2})}{k_2' k_{i3}},\ c_3 = 1.$$

For the case when we have in the first internal contour *PI* controller we will have the follow equivalent transfer function of the object (8) with *PI* controller in the first internal contour and t.f. of subprocess $H_{F2}(s)$ (2)

$$H_{\Sigma F2}(s) = H'_{F3}(s) H_{F2}(s) = \frac{b_0 s + b_1}{a_0 s^3 + a_1 s^2 + a_2 s + a_3}, \quad (15)$$

where $b_0 = k_2 d_0 = \frac{k_{p2}}{k_{i2}} k_2,\ b_1 = k_2 d_1 = k_2,\ a_0 = c_0 T_2 = \frac{T_2 T_3}{k_2 k_{i2}},$

$$a_1 = c_0 + c_1 T_2 = \frac{T_3 + T_2(1 + k_3 k_{p3})}{k_3 k_{i3}},\ a_2 = c_1 + c_2 T_2 =$$

$$\frac{1 + k_3 k_{p3}}{k_3 k_{i2}} + T_2 = \frac{1 + k_3(k_{p2} + T_2 k_{i2})}{k_3 k_{i2}};\ a_3 = 1.$$

- ***P* controller** is tuning to the object with transfer function (15), applied the M.S.D. method and tuning parameters of controller are determinated from relations [4,5,6]

$$k_{p2} = \frac{a_0 J^3 - a_1 J^2 + a_2 J - a_3}{b_1 - b_0 J}. \quad (16)$$

In the relation (16) J is the maximal stability degree and which is chose from the following condition $J>0$. The t.f. of the second internal contour with *P* controller

$$H'_{F2}(s) = \frac{H_{R2}(s) H_{F2}(s)}{1 + H_{R2}(s) H_{F2}(s)} = \frac{d_0 s + d_1}{c_0 s^3 + c_1 s^2 + c_2 s + c_3}, \quad (17)$$

where

$$d_0 = \frac{k_{p2} b_0}{a_3 + k_{p2} b_1},\ d_1 = \frac{k_{p2} b_1}{a_3 + k_{p2} b_1},\ c_0 = \frac{a_0}{a_3 + k_{p2} b_1},$$

$$c_1 = \frac{a_1}{a_3 + k_{p2} b_1},\ c_2 = \frac{a_2 + k_{p2} b_0}{a_3 + k_{p2} b_1},\ c_3 = 1.$$

***PI* controller** is tuning to the object with the transfer function (15), applied the M.S.D. method and tuning parameters of controller are determinated from relations [4,5,6]

$$k_{p2} = \frac{-3a_0 b_0 J^4 + J^3(4a_0 b_1 + 2a_1 b_0) - J^2(3a_1 b_1 + a_2 b_0) + 2a_2 b_1 J - a_3 b_1}{(b_1 - b_0 J)^2}, \quad (18)$$

$$k_{i2} = \frac{-a_0 J^4 + a_1 J^3 - a_2 J^2 + a_3 J}{(b_1 - b_0 J)} + k_{p2} J. \quad (19)$$

We can obtain the values of parameters k_{p2}, k_{i2}, changing the $J>0$ value, for that the performances of control system are predefined.

The t.f. of internal contour with *PI* controller is

$$H'_{F2}(s) = \frac{H_{R2}(s) H_{F2}(s)}{1 + H_{R2}(s) H_{F2}(s)} = \frac{d_0 s^2 + d_1 s + d_2}{c_0 s^4 + c_1 s^3 + c_2 s^2 + c_3 s + c_4}, \quad (20)$$

where $d_0 = \frac{k_{p2} b_0}{k_{i2} b_1},\ d_1 = \frac{k_{p2} b_1}{k_{i2} b_1},\ d_2 = 1,\ c_0 = \frac{a_0}{k_{i2} b_1},\ c_1 = \frac{a_1}{k_{i2} b_1},$

$$c_2 = \frac{a_2 + k_{p2} b_0}{k_{i2} b_1},\ c_3 = \frac{a_3 + k_{p2} b_1 + k_{i2} b_0}{k_{i2} b_1},\ c_4 = 1.$$

2.3 The tuning controllers in the external contour

The structure block scheme of the *external contour* is represented in the figure 3 *a, b*.

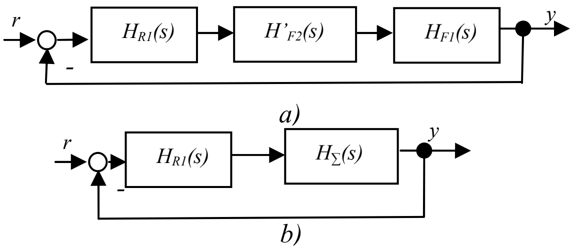

Fig.3 - Structure block schema of external contour

Tuning of *P, PI, PID* controllers in the external contour it was made for four case, when in the first internal contour it was tuning *P* and *PI* controller and in the second internal contour in was tuning *P* and *PI* controllers.

I case. In the first case it is necessary to determine the equivalent transfer function of the object (11) with *P* controller in the first internal contour and *P* controller in the second internal contour with the transfer function of subprocess $H_{F1}(s)$ (1)

$$H_{\Sigma}(s) = H'_{F2}(s) H_{F1}(s) = \frac{k}{a_0^* s^3 + a_1^* s^2 + a_2^* s + a_3^*}. \quad (21)$$

where $a_0^* = a_0' T_1,\ a_1^* = a_1' T_1 + a_0,\ a_2^* = a_2' + a_1',\ a_3^* = a_2',\ k = k_1 k_2^*.$

For object with t.f. (21) *P, PI, PID* controllers can be tune applied the M.S.D. method using the relation from [4,5,6]:

Control system with P controller:

$$k_{p1} = \frac{1}{k}(a_0^* J^3 - a_1^* J^2 + a_2^* J - a_3^*). \quad (22)$$

Control system with PI controller:

$$k_{p1} = \frac{1}{k}(4a_0^* J^3 - 3a_1^* J^2 + 2a_2^* J - a_3^*). \quad (23)$$

$$k_{i1} = \frac{J^2}{k}(3a_0^* J^2 - 2a_1^* J + a_2^*). \quad (24)$$

Control system with PID controller:

$$k_{d1} = \frac{1}{k}(-6a_0^* J^2 + 3a_1^* J - a_2^*), \quad (25)$$

$$k_{p1} = \frac{1}{k}(4a_0^* J^3 - 3a_1^* J^2 + 2a_2^* J - a_3^*) + 2k_d J. \quad (26)$$

$$k_{i1} = \frac{1}{k}(-a_0^* J^4 + a_1^* J^3 - a_2^* J^2 + a_3^* J) - k_d J^2 + k_p J. \quad (27)$$

Values of k_{p1}, k_{i1}, k_{d1} parameters are obtained, made the variation of J value, for that the performances of control system are predefined.

II case. In the second case it is necessary to determine the equivalent transfer function of the object (14) with *P* controller in the first internal contour and *PI* controller in the second internal contour with the transfer function of subprocess $H_{F1}(s)$ (1)

$$H_\Sigma(s) = H'_{F2}(s) H_{F1}(s) = \frac{b_0 s + b_1}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}, \quad (28)$$

where $b_0 = k_1 d_0$, $b_1 = k_1 d_1$, $a_0 = c_0 T_1$, $a_1 = c_1 T_1 + c_0$,

$a_2 = c_2 T_1 + c_1$, $a_3 = c_3 T_1 + c_2$, $a_4 = 1$.

For object with t.f. (28) *P, PI, PID* controllers can be tune applied the M.S.D. method using the relation from [4,5,6]:

Control system with P controller:

$$k_{p1} = \frac{-a_0 J^4 + a_1 J^3 - a_2 J^2 + a_3 J - a_4}{b_1 - b_0 J}. \quad (29)$$

Control system with PI controller:

$$k_{p1} = \frac{d_0 J^5 - d_1 J^4 + d_2 J^3 - d_3 J^2 + d_4 J - d_5}{(b_1 - b_0 J)^2}, \quad (30)$$

where

$d_0 = 4a_0 b_0^3$; $d_1 = 5a_0 b_1 + 3a_1 b_1$; $d_2 = 4a_1 b_1 + 2a_2 b_0$;

$d_3 = 3a_2 b_1 + a_3 b_0$; $d_4 = 2a_3 b_1$; $d_5 = b_1 a_4$;

$$k_{i1} = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2 + a_4 J}{b_1 - b_0 J} + k_{p1} J. \quad (31)$$

Control system with PID controller:

$$k_{d1} = \frac{-d_0 J^6 + d_1 J^5 - d_2 J^4 + d_3 J^3 - d_4 J^2 + d_5 J - d_6}{2(b_1 - b_0 J)^4}, \quad (32)$$

where $d_0 = 12 a_0 b_0^3$; $d_1 = 42 a_0 b_0^2 b_1 + 6 a_1 b_0^3$;

$d_2 = 50 a_0 b_0 b_1^2 + 22 a_1 b_0^2 b_1 + 2 a_2 b_0^3$;

$d_3 = 20 a_0 b_1^3 + 28 a_1 b_0 b_1^2 + 8 a_2 b_0^2 b_1$;

$d_4 = 12 a_1 b_1^3 + 12 a_2 b_0 b_1^2$; $d_5 = 6 a_2 b_1^3 +$

$+ 2 a_3 b_0 b_1^2 - 2 b_0^2 b_1$; $d_6 = 2 a_3 b_1^3 - 2 b_0 b_1^2$;

$$k_{p1} = \frac{\left(d_0 J^5 - d_1 J^4 + d_2 J^3 - d_3 J^2 + d_4 J - d_5\right)}{\left(b_1 - b_0 J\right)^2} + 2 k_{d1} J, \quad (33)$$

where $d_0 = 4a_0 b_0$; $d_1 = 5a_0 b_1 + 3a_0 b_1$; $d_2 = 4a_1 b_1 + 2a_2 b_0$;

$d_3 = 3a_2 b_1 + a_3 b_0$; $d_4 = 2a_3 b_1$; $d_5 = a_4 b_1$;

$$k_{i1} = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2 + J}{b_1 - b_0 J} - k_{d1} J^2 + k_{p1} J. \quad (34)$$

Values of k_{p1}, k_{i1}, k_{d1} parameters are obtained, made the variation of J value, for that the performances of control system are predefined.

III case. In the third case it is necessary to determine the equivalent transfer function of the object (17) with *PI* controller in the first internal contour and *P* controller in the second internal contour with the transfer function of subprocess $H_{F1}(s)$ (1)

$$H_\Sigma(s) = H'_{F2}(s) H_{F1}(s) = \frac{b_0 s + b_1}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}, \quad (35)$$

where $b_0 = k_1 d_0$, $b_1 = k_1 d_1$, $a_0 = c_0 T_1$, $a_1 = c_1 T_1 + c_0$,

$a_2 = c_2 T_1 + c_1$, $a_3 = c_3 T_1 + c_2$, $a_4 = 1$.

For object with t.f. (35) *P, PI, PID* controllers can be tuned applying the M.S.D. method using the relation from [4,5,6]:

Control system with P controller:

$$k_{p1} = \frac{-a_0 J^4 + a_1 J^3 - a_2 J^2 + a_3 J - a_4}{b_1 - b_0 J}. \quad (36)$$

Control system with PI controller:

$$k_{p1} = \frac{d_0 J^5 - d_1 J^4 + d_2 J^3 - d_3 J^2 + d_4 J - d_5}{(b_1 - b_0 J)^2}, \quad (37)$$

where $d_0 = 4a_0 b_0$; $d_1 = 5a_0 b_1 + 3a_1 b_1$; $d_2 = 4a_1 b_1 + 2a_2 b_0$;

$d_3 = 3a_2 b_1 + a_3 b_0$; $d_4 = 2a_3 b_1$; $d_5 = b_1 a_4$;

$$k_{i1} = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2 + a_4 J}{b_1 - b_0 J} + k_{p1} J. \quad (38)$$

Control system with PID controller:

$$k_{d1} = \frac{-d_0 J^6 + d_1 J^5 - d_2 J^4 + d_3 J^3 - d_4 J^2 + d_5 J - d_6}{2(b_1 - b_0 J)^4}, \quad (39)$$

where $d_0 = 12 a_0 b_0^3$; $d_1 = 42 a_0 b_0^2 b_1 + 6 a_1 b_0^3$;

$d_2 = 50 a_0 b_0 b_1^2 + 22 a_1 b_0^2 b_1 + 2 a_2 b_0^3$; $d_3 = 20 a_0 b_1^3 + 28 a_1 b_0 b_1^2 + 8 a_2 b_0^2 b_1$;

$d_4 = 12 a_1 b_1^3 + 12 a_2 b_0 b_1^2$; $d_5 = 6 a_2 b_1^3 + 2 a_3 b_0 b_1^2 - 2 b_0^2 b_1$; $d_6 = 2 a_3 b_1^3 - 2 b_0 b_1^2$;

$$k_{p1} = \frac{\left(d_0 J^5 - d_1 J^4 + d_2 J^3 - d_3 J^2 + d_4 J - d_5\right)}{\left(b_1 - b_0 J\right)^2} + 2 k_{d1} J, \quad (40)$$

where $d_0 = 4a_0 b_0$; $d_1 = 5a_0 b_1 + 3a_0 b_1$; $d_2 = 4a_1 b_1 + 2a_2 b_0$;

$d_3 = 3a_2 b_1 + a_3 b_0$; $d_4 = 2a_3 b_1$; $d_5 = a_4 b_1$;

$$k_{i1} = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2 + J}{b_1 - b_0 J} - k_{d1} J^2 + k_{p1} J. \quad (41)$$

Values of k_{p1}, k_{i1}, k_{d1} parameters are obtained, made the variation of J value, for that the performances of control system are predefined.

IV case. In the forth case it is necessary to determine the equivalent transfer function of the object (20) with *PI* controller in the first internal contour and *PI* controller in the second internal contour with the transfer function of subprocess $H_{F1}(s)$ (1)

$$H_\Sigma(s) = H'_{F2}(s) H_{F1}(s) = \frac{b_0 s^2 + b_1 s + b_2}{a_0 s^5 + a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s + a_5}, \quad (42)$$

where $b_0 = k_1 d_0$, $b_1 = k_1 d_1$, $b_2 = k_1 d_2$, $a_0 = c_0 T_1$, $a_1 = c_1 T_1 + c_0$,

$a_2 = c_2 T_1 + c_1$, $a_3 = c_3 T_1 + c_2$, $a_4 = T_1 c_4 + c_3$, $a_5 = 1$.

For object with t.f. (42) *P, PI, PID* controllers can be tune applied the M.S.D. method using the relation from [4,5,6]:

Control system with P controller:

$$k_{p1} = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2 + a_4 J - a_5}{b_0 J^2 - b_1 J + b_2}. \qquad (43)$$

Control system with PI controller:

$$k_{p1} = \frac{d_0 J^7 - d_1 J^6 + d_2 J^5 - d_3 J^4 + d_4 J^3 - d_5 J^2 + d_6 J - d_7}{(b_0 J^2 - b_1 J + b_2)^2}, \qquad (44)$$

where $d_0 = 4a_0 b_0$; $d_1 = 5a_0 b_1 + 3a_1 b_0$; $d_2 = 6a_0 b_2 + 4a_1 b_1 + 2a_2 b_0$; $d_3 = 5a_1 b_2 + 3a_2 b_1 + a_3 b_0$; $d_4 = 4a_2 b_2 + 2a_3 b_1$; $d_5 = 3a_3 b_2 + a_4 b_1 - a_5 b_0$; $d_6 = 2a_4 b_2$; $d_7 = a_5 b_2$.

$$k_{i1} = \frac{-a_0 J^6 + a_1 J^5 - a_2 J^4 + a_3 J^3 - a_4 J^2 + a_5 J}{b_0 J^2 - b_1 J + b_2} + k_p J. \qquad (45)$$

Control system with PID controller:

$$k_{d1} = \frac{-d_0 J^{10} + d_1 J^9 - d_2 J^8 + d_3 J^7 - d_4 J^6 + d_5 J^5 - d_6 J^4 + d_7 J^3 - d_8 J^2 + d_9 J - d_{10}}{2(b_0 J^2 - b_1 J + b_2)^4}, \qquad (46)$$

where $d_0 = 2a_0 b_0^3$; $d_1 = 42a_0 b_0^2 b_1 + 6a_1 b_0^3$;

$d_2 = 46a_0 b_0^2 b_2 + 80a_0 b_0 b_1^2 + 40a_1 b_0^2 b_1 + 2a_2 b_0^3$;

$d_3 = 100a_0 b_0 b_1 b_2 + 20a_0 b_1^3 + 28a_1 b_0 b_1^2 + 24a_1 b_0^2 b_2 + 8a_2 b_0^2 b_1$;

$d_4 = 64a_0 b_0 b_2^2 + 68a_0 b_1^2 b_2 + 64a_1 b_0 b_1 b_2 + 12a_1 b_1^3 + 8a_2 b_0^2 b_2 + 12a_2 b_0 b_1^2$;

$d_2 = 46a_0 b_0^2 b_2 + 80a_0 b_0 b_1^2 + 40a_1 b_0^2 b_1 + 2a_2 b_0^3$;

$d_3 = 100a_0 b_0 b_1 b_2 + 20a_0 b_1^3 + 28a_1 b_0 b_1^2 + 24a_1 b_0^2 b_2 + 8a_2 b_0^2 b_1$;

$d_4 = 64a_0 b_0 b_2^2 + 68a_0 b_1^2 b_2 + 64a_1 b_0 b_1 b_2 + 12a_1 b_1^3 + 8a_2 b_0^2 b_2 + 12a_2 b_0 b_1^2$;

$d_5 = 78a_0 b_1 b_2^2 + 48a_0 b_0 b_2^2 + 52a_1 b_1^2 b_2 + 32a_1 b_0 b_1 b_2 + 16a_2 b_0 b_2 (b_1 + b_0) + 12a_2 b_1^3 +$
$+ 4a_3 b_0 b_1^2 - 12a_3 b_0 b_2^2 - 6a_3 b_0^2 b_2 - 6a_3 b_1^2 b_2 - 4a_4 b_0 b_1 b_2 - 2a_4 b_0^2 b_1 - 2a_4 b_1^3 +$
$+ 4a_5 b_0^2 b_2 + 2a_5 b_0^3 + 2a_5 b_0 b_1^2$; $d_6 = 30a_0 b_2^3 + 50a_1 b_1 b_2^2 + 18a_2 b_2 (b_0 b_2 + b_1^2) +$
$+ 2a_3 b_1 (2b_0 b_2 + b_1^2) - 2a_4 b_0 (b_1^2 + 2b_0 b_2) + 2a_5 b_0^2 b_1$;

$d_7 = 20a_1 b_2^3 + 28a_2 b_1 b_2^2 + 4a_3 b_0 b_2^2 + 8a_3 b_1^2 b_2 - 8a_4 b_0 b_1 b_2 - 4a_5 b_0^2 b_2$;

$d_8 = 12a_2 b_2^3 + 6a_3 b_2^2 (b_1 + b_0) + 2a_4 b_0 b_1 b_2 - 4a_4 b_0 b_2^2 - 2a_4 b_0 b_2^2 - 2a_5 b_0^2 b_2 -$
$- 6a_5 b_0 b_1 b_2$; $d_9 = 12a_3 b_0 b_2^2 + 6a_3 b_1^2 b_2 + 4a_4 b_0 b_1 b_2 + 2a_4 b_1^3 -$
$- 4a_5 b_0 b_2 (b_0 + b_2) - 2a_5 b_1^2 (b_0 + b_2)$; $d_{10} = 2a_4 b_2^3 - 2a_5 b_1 b_2^2$.

$$k_{p1} = \frac{d_0 J^7 - d_1 J^6 + d_2 J^5 - d_3 J^4 + d_4 J^3 - d_5 J^2 + d_6 J - d_7}{(b_0 J^2 - b_1 J + b_2)^2} + 2k_{d1} J, \qquad (47)$$

where $d_0 = 4a_0 b_0$; $d_1 = 5a_0 b_1 + 3a_1 b_0$; $d_2 = 6a_0 b_2 + 4a_1 b_1 + 2a_2 b_0$; $d_3 = 5a_1 b_2 + 3a_2 b_1 + a_3 b_0$; $d_4 = 4a_2 b_2 + 2a_3 b_1$; $d_5 = 3a_3 b_2 + a_4 b_1 - a_5 b_0$; $d_6 = 2a_4 b_2$; $d_7 = a_5 b_2$.

$$k_{i1} = \frac{-a_0 J^6 + a_1 J^5 - a_2 J^4 + a_3 J^3 - a_4 J^2 + a_5 J}{b_0 J^2 - b_1 J + b_2} - k_{d1} J^2 + k_{p1} J. \qquad (48)$$

Values of k_{p1}, k_{i1}, k_{d1} parameters are obtained, made the variation of *J* value, for that the performances of control system are predefined.

3. APPLICATION AND COMPUTER SIMULATION

To show the efficiency of the proposed algorithm for tuning of the typical controllers in the multiple-loop feedback control system with inertia (third order) using the presented relations was examine an example with the object models which has the following parameters: $H_{F3}(s)$: $k_3=0.1$, $T_3=0.5$, $H_{F2}(s)$: $k_2=0.5$, $T_2=1$ and $H_{F1}(s)$: $k_1=2$, $T_1=2$.

The *P, PI* controllers were tuning in the first and second internal contour using the maximal stability degree method, which permitted to obtain the high performance, varied values of *J* and choosing the k_{p3} and k_{p3}, k_{i3}; k_{p2} and k_{p2}, k_{i2} values for respectively controllers.

The *P, PI* and *PID* controllers were tuning in external contour using the maximal stability degree method, which permitted to obtain the high performance, varied values of *J* and choosing the respectively values of the *P, PI, PID* controllers.

The computer simulation have been made in MATLAB and the simulation diagram of multiple-loop feedback control system is presented in figure 4.

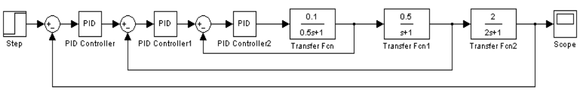

Fig. 4 - Simulation diagrams of the control system

In the fig. 5 are presented the transition processes of the multiple-loop feedback control system for external contour: *a)* – transition process in the external contour: with *P* controller in the first internal contour ($k_{p3}=3.5$) and *P* controller in the second internal controller ($k_{p2}=2.064$): external contour with *P* controller ($k_{p1}=0.3966$) - curve *1*; with *PI* controller with optimal parameters ($J_{opt}=0.58$, $k_{p1}=1.033$, $k_{i1}= 0.5178$) – curve *2*; with *PID* controller with optimal parameters ($J_{opt}=1.05$, $k_{p1}=3.188$, $k_{i1}= 1.327$, $k_{d1}= 1.4662$) – curve *3*, optimization in Matlab - curves 4*; b)* – transition process in the external contour with *P* controller in the first internal contour ($k_{p3}=3.5$) and *PI* controller in the second internal controller ($k_{p2}=0.1977$, $k_{i2}= 2.236$): external contour with *P* controller ($k_{p1}=0.00826$) - curve *1*; with *PI* controller with optimal parameters ($J_{opt}=0.35$, $k_{p1}=0.095$, $k_{i1}= 0.0488$) – curve *2*; with *PID* controller with optimal parameters ($J_{opt}=0.35$, $k_{p1}=0.1121$, $k_{i1}= 0.051$, $k_{d1}= 0.0238$) – curve *3*, optimization in Matlab - curves 4*; c)* – transition process in the external contour with *PI* controller in the first internal contour ($k_{p3}=0.2$, $k_{i3}= 5.202$) and *P* controller in the second internal controller ($k_{p2}=0.00128$): external contour with *P* controller ($k_{p1}=11.39$) - curve *1*; with *PI* controller with optimal parameters ($J_{opt}=0.33$, $k_{p1}=196.263$, $k_{i1}= 92.044$) – curve *2*; with *PID* controller with optimal parameters ($J_{opt}=051$, $k_{p1}=497.546$, $k_{i1}= 150.39$, $k_{d1}= 393.531$) – curve *3*, optimization in Matlab - curves *4; d)* – transition process in the external contour with *PI* controller in the first internal contour ($k_{p3}=0.2$, $k_{i3}= 5.202$) and *PI* controller in the second internal controller ($k_{p2}=0.5027$, $k_{i2}= 0.3837$): external contour with *P* controller ($k_{p1}=0.41$) - curve *1*; with *PI* controller with optimal parameters *(J_{opt}=0.22, k_{p1}=0.1136, k_{i1}= 0.0385)* – curve *2*; with *PID* controller with optimal parameters *(J_{opt}=0.1, k_{p1}=0.138, k_{i1}= 0.028, k_{d1}=0.80209)* – curve *3,* optimization in Matlab - curves 4.

In the fig. 6 are presented the pole zero maps for follow case: *a)* when in the first and second internal contour was tuning *P* controller and in the extetrnal contour was tuning *P* – note *1, PI* – note *2, PID*- note *3* controllers; *b)* when in the first internal contour was tuning *P* controller, in the second internal contour was tuning *PI* controller and in the extetrnal contour was tuning *P* – note *1, PI* – note *2, PID*- note *3* controlers; *c)* when in the first internal contour was tuning *PI* controller, in the second internal contour was tuning *P* controller and in the extetrnal contour was tuning *P* – note *1, PI* – note *2, PID*- note *3* controlers; *d)* when in the first and second internal contour was tuning *PI* controller and in the extetrnal contour was tuning *P* – note *1, PI* – note *2, PID*- note *3* controllers.

In the fig. 7 are presented the transition processes for the cases when the parametres of object are varied: *a)* when in the first, second internal contour was tuning *P* controller and in the external contour was tuning *PID* controller; *b)* when in the first internal contour was tuning *P* controller, in the second internal contour was tuning *PI* controller and in the extetrnal contour was tuning *PID; c)* when in the first internal contour was tuning *PI* controller, in the second internal contour was tuning *P* controller and in the extetrnal contour was tuning *PID; d)* when in the first internal contour was tuning *PI* controller, in the second internal contour was tuning *PI* controller and in the extetrnal contour was tuning *PID.* For these cases we have curve 1 – the original curve; curves *2,3* -variation of T_1, T_2, T_3 with *±20%;* curves *4,5* -variation of k_1, k_2, k_3 with *±20%; 6,7* -variation of T_1, T_2, T_3 with *±30%.*

Analysing the obtained results we can mention from fig. 5 *a, b, c, d* the following: tuning the *PI* and *PID* controller, using the maximal stability degree method the controllers' parameters get the optimal values.

CONCLUSIONS

As a results of obtained results after tuning the *P, PI, PID* controller to the multiple-loop feedback control system with object's models (1), (2), (3) with known parameters, the following conclusion can be made:

1. The tuning of *P, PI* controllers in the internal contour in conformity with the maximal stability degree method permittes to obtain the high results varying the value of the *J>0* and choosing the parameters of the respectively controllers for obtain the predefined performance of the internal contour. For the *PI* controller it is obtained the optimal values.
2. The tuning of *P, PI, PID* controllers in the external contour using the maximal stability degree method permittes to obtain the high results varying the value of the *J>0* and obtained the optimal value and the suboptimal values of the *J*, and choosing the set of the values of regulator's parameters for obtain the predefined performances.

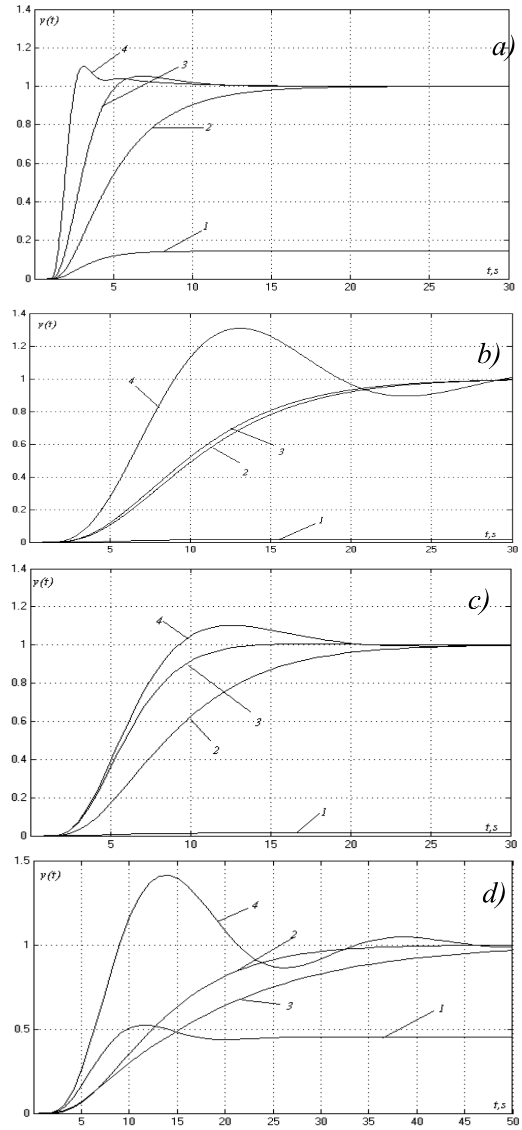

Fig. 5 - Transition processes of the multiple-loop feedback control system

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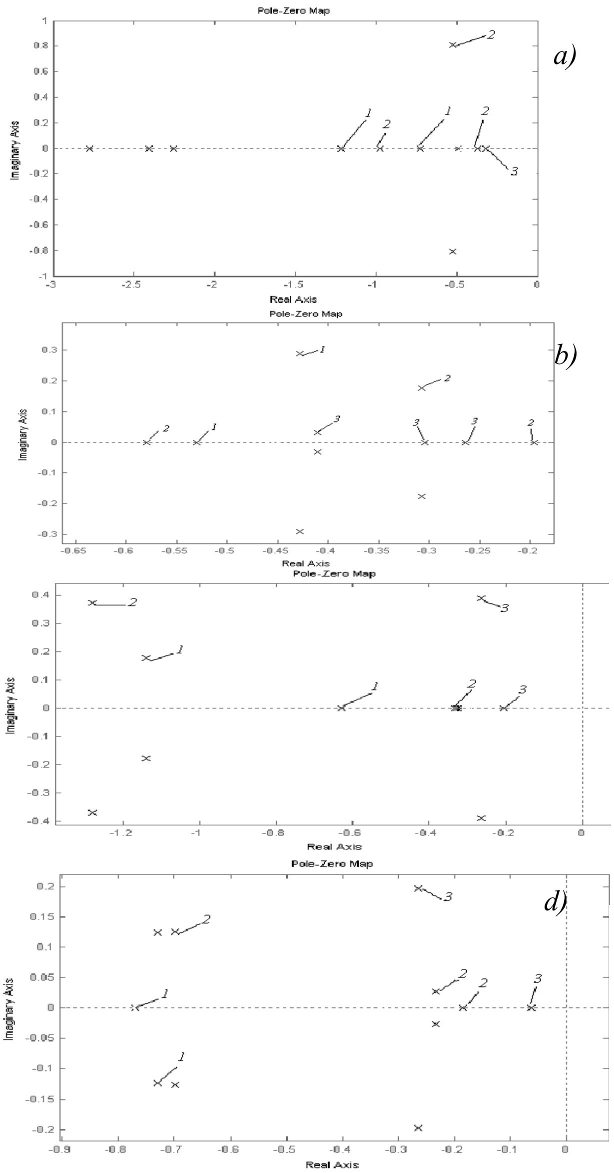

Fig. 6 - The pole – zero map

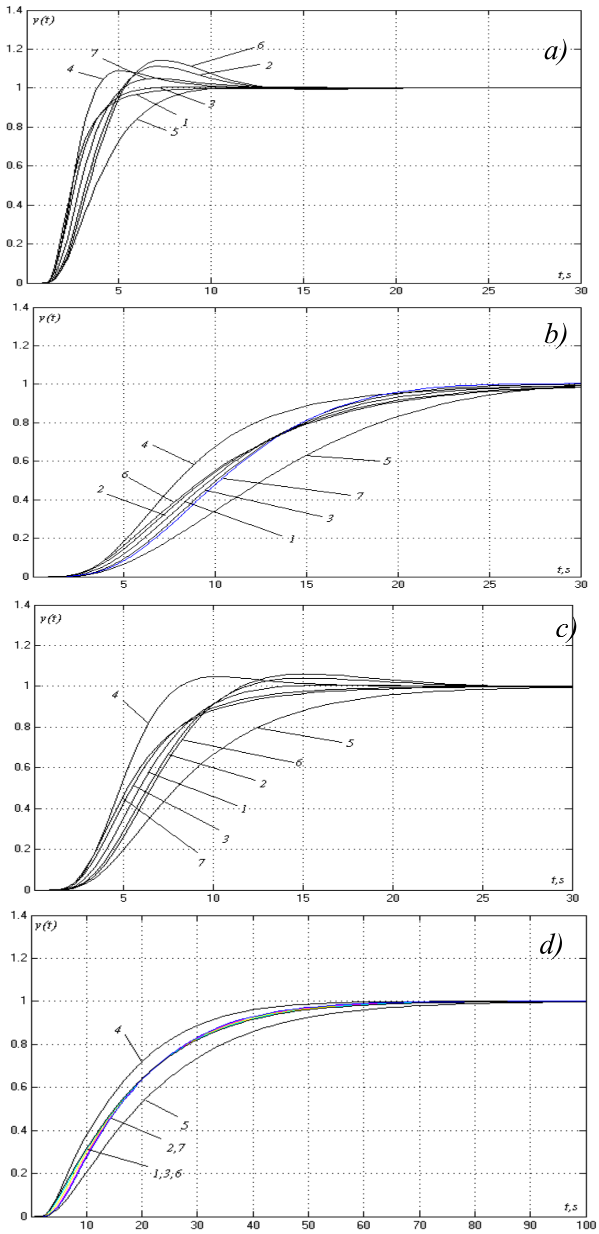

Fig. 7 - Transition processes of the multiple-loop feedback control system in the case of the variation parameters of the control object

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