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GEOMETRICAL ASPECTS OF THE COVARIANT DYNAMICS OF HIGHER ORDER

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Abstract. We present some geometrical aspects of a higher-order jet bundle which is considered a suitable framework for the study of higher-order dynamics in continuous media. We generalize some results obtained by A. Vondra, [7]. These results lead to a description of the geometrical dynamics of higher order generated by regular equations.

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Introduction

The present study is an attempt to emphasize some geometrical aspects of a possible mathematical model for the higher-order dynamics in continuous media as well as for the higher-order field theories.

The mathematicians agree (see [1], [2], [4], etc) that the most suitable framework for this application is a higher-order jet bundle associated to a fibered manifold. A physical field is a section of this "configuration manifold". The partial differential equations describing some higher-order dynamics are the kernels of some operators which appear as sections in a vector bundle of forms over that jet bundle, [1].

A. Vondra initiated such a study for a fibered manifold having the base of dimension 1, [5], [6], [7].

We consider a fibered manifold (E, π_0, B), where B is an orientable manifold of dimension $n \geqslant 1$ ("parameter space" containing $n-1$ "spatial variables" and a "time variable"), E is a manifold of dimension $n+m$ and π_0 is a submersion of E on B.

In [4] one argues the importance of a covariant approach that is the time variable and the other parameters on the whole.

To start the study it is necessary to define some associated structures and geometrical objects as $f(3,-1)$-structures, contact forms, connection of order r, dynamical connections.

Our approach means, in a more general context, to consider the $f(3,-1)$-structure on a jet bundle introduced by Vondra in the case $n=1$, [6].

The results of § 4 ($r=1$) generalize those obtained by Vondra in [7]. These results lead to a description of the geometrical dynamics of higher order generated by regular equations.

We shall use the standard multi-index notation. A multi-index is denoted by $I=(i_1,\dots,i_n)\in\mathbb{N}^n$. The length of I is $|I|=i_1+\dots+i_n$ and its power is $w(I)=|I|!/I!$ where $I!=i_1!\dots i_n!$. $0=(0,\dots,0)$ is the null multi-index and $1_i=(0,\dots,1,\dots 0)$ with 1 at the i-th place. For $I=(i_1,\dots,i_n)$, $J=(j_1,\dots,j_n)$ we define the sum $I+J=(i_1+j_1,\dots,i_n+j_n)$. In particular, $Ij=jI=I+1_j=(i_1,\dots,i_{j-1},i_j+1,i_{j+1},\dots,i_n)$. For a family of objects $A=\{a_{i,J},|I|=m,|J|=1\}$ with m,l fixed, we may define a new family $\sigma(A)=\{\sigma_L(A),|L|=m+1\}$ by

$$\sigma_L(A)=\frac{1}{w(L)}\sum_{I+J=L}w(I)w(J)a_{I,J}$$

(the sum is made for all multi-indexes I,J with $I+J=L$). The family of objects $A=\{a_{I,j},|I|=m\}$ is identified with the family $A=\{a_{I,1_j},|I|=m\}$ for which $\sigma(A)=\{\sigma_L(A),|L|=m+1\}$, where

$$\sigma_L(A)=\frac{1}{w(L)}\sum_{I+J=L}w(I)a_{I,J},\ |I|=m,\ |J|=1.$$

All manifolds and mappings are supposed to be smooth and the summation convention is used as far as possible.

1. Geometric structures on J^pE

Let (E,π_0,B) be a fibered manifold with dim $B=n$, dim $E=n+m$, (U,x^i) a local chart on B and $(U_0=\pi_0^{-1}(U),x^i,u^\alpha)$ the local fibered chart on E adapted to (U,x^i). If $(\overline{U}_0,\bar{x}^i,\bar{u}^\alpha)$ is another chart local fibered charts on E adapted to $(\overline{U},\bar{x}^i)$ and $U\cap\overline{U}\neq\emptyset$ then the coordinate transformations are

$$\bar{x}^i=\bar{x}^i(x),\ \det\|\overline{B}^i_j\|\neq 0,\ \overline{B}^i_j=\frac{\partial\bar{x}^i}{\partial x^j};\tag{1.1}$$

$$\bar{u}^\alpha=\bar{u}^\alpha(x,u),\ \det\|\overline{A}^\alpha_\beta\|\neq 0,\ \overline{A}^\alpha_\beta=\frac{\partial\bar{u}^\alpha}{\partial u^\beta}.$$