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Graphical method of solving problems on bistability in physical systems

Vitalie Chistol and Vasile Tronciu*

Department of Physics, Technical University of Moldova, Stefan cel Mare av. 168, Chisinau MD-2004, Moldova

E-mail: vasile.tronciu@fiz.utm.md

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Abstract

Solving physics problems is often done superficially, without making an in-depth analysis of the results obtained, limiting itself only to determining the numerical values of some physical quantities. In order to analyze the results obtained after solving physics problems and to more deeply understand the essence of the phenomena that take place, a graphic representation of the obtained results is often necessary. In this paper, we present examples of solving different problems from physical systems for which the effect of bistability occurs. We use the graphical method to analyze the results obtained. We mention that the results presented in this paper are suited for teaching undergraduate students.

Keywords: graphical method, bistability, mechanics, undergraduate students, physical systems

(Some figures may appear in colour only in the online journal)

1. Introduction

The phenomenon of bistability is well-known in optics (optical bistability) [1], magnetism (magnetic bistability) [2], electricity (bistable circuits) [3] etc. In most cases the nature of this phenomenon is quantum. We propose in this paper an analysis of two examples of bistability in mechanics, and relatively simple explanation of this phenomenon. We use the graphical method to synthesize the results predicted by the theoretical models. Two examples could be useful for students to understand the phenomenon of bistability in physics.

The paper is structured as follows. We start in section 2 by describing rotation of cylinder with mobile piston. Section 3 shows bistable behavior of displacement of mercury in tubes.

*Author to whom any correspondence should be addressed.

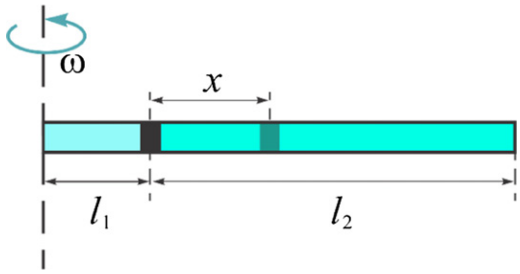

Figure 1. The scheme of the cylinder with mobile piston inside. By black, we show the initial position of piston when cylinder is static and by gray when the cylinder is rotating.

These two setups are well explained by graphical method. The summary and conclusions are given in section 4.

2. Rotation of cylinder with mobile piston

Let start our analysis considering a horizontal cylinder closed at both sides with a mobile piston of mass m inside. The piston can slide without friction along the cylinder.

The pressure of air in both sides of the cylinder is p_0, when cylinder is static in initial position. The initial distances from center of piston to the ends of cylinder are respectively l_1 and l_2 (see figure 1). We consider the follow question: at which angular velocity has to be rotated the cylinder so that the displacement of the piston from the equilibrium position is x?

To solve this problem, we find the forces acting to the piston. Two forces act to the piston from left and right parts of cylinder with the result force F_1 of the following form

$$F_1 = (p_2 - p_1)S, \tag{1}$$

where p_1 and p_2 are respectively, the air pressures on the left and right sides of piston during its rotation. S is the cross-sectional area of the cylinder. We mention that, during the rotation of the cylinder this force always is oriented to the axis of rotation.

In what follows, we consider the air in both sides of the cylinder subject to an isothermal process. Thus, we have

$$p_0 l_1 S = p_1(l_1 + x)S, \qquad p_0 l_2 S = p_2(l_2 - x)S.$$

Expressing p_1 and p_2 from the last equations and introducing them in (1), we obtain

$$F_1 = \left(\frac{p_0 l_2}{l_2 - x} - \frac{p_0 l_1}{l_1 + x}\right) S. \tag{2}$$

The force that gives to the piston centripetal acceleration can be written in the form

$$F_2 = m\omega^2(l_1 + x), \tag{3}$$

where ω is angular velocity.

When the piston is in equilibrium $F_1 = F_2$, i.e.

$$\left(\frac{p_0 l_2}{l_2 - x} - \frac{p_0 l_1}{l_1 + x}\right) S = m\omega^2(l_1 + x).$$