Synthesis Algorithms of Controllers for Automatic Control Systems with Maximum Stability Degree

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***Abstract* - Algorithms for synthesis of the P, PD, PI and PID controllers for control systems with maximum stability degree are elaborated in this paper. The imposing to the designed system of the maximum stability degree is provided by the following statement: the maximum possible stability degree of the automatic control system can be achieved when the real parts of all roots of the characteristic equation are equal. The proposed algorithms represent a simple procedure, that are reduced to use of algebraic equations and can be applied to the objects' models with inertia of arbitrary order, astatism and time delay. In comparison with other methods of controllers' synthesis, the proposed algorithms provide good performance to the designed automatic control systems, resulting the aperiodic transient processes. They allow also to impose or to optimize the settling time of the designed automatic control system. These algorithms may be used both for synthesis of the conventional automatic systems and for synthesis of the automatic systems with auto tuning and the adaptive systems.**

Keywords - *automatic control systems, control object, objects' models with inertia and time delay, controllers, synthesis of controllers, maximum stability degree.*

I. INTRODUCTION

Most of controllers, that are in current operation in the industrial installations, using the PID algorithm and its variations, thanks to its simplicity and good performance offered to the designed automatic control systems. But in regard to determining the optimal values of the dynamic tuning parameters of PID controllers in order to achieve a better functioning of the automatic control systems, it is noted difficulties.

There is a variety of methods for synthesis of controllers [1, 2, 3]. Each method has its specifics in terms of the applicability and the control quality. As a rule, most methods only provide the stability of automatic system, but do not guarantee the satisfaction of some good performance of the control process.

The analytical methods, for example Coon, Shedel, the method of the module, the method of the symmetry and so on, are based on the approximation of the dynamic process with low inertia models, and only some of them, as would be the method of the optimum amplitude, use the high degree models. This is explained by the fact that there are difficulties in the analytical solution of the equation systems which describe the PID controller for the high order models of objects.

The graphic-analytical methods, such as roots locus method and frequency method, offer satisfactory performance of the designed system, but require a large volume of calculations and graphic construction.

The experimental tuning methods such as Ziegler-Nichols, Chien-Hrones-Reswich, Offerens and so on, are simple to use, but do not allow the imposition of the desired performance and the quality of control by these methods is often unsatisfactory and requires additional adjustment. In the case of the Ziegler-Nichols method the system is brought to the limit of stability, which in some cases imposes to the system a negative mode of operation.

The parametric optimization method is based on the integral criteria and characterizes the global behavior of the automatic system. It can be applied to the complex objects' models, provides good performance to the designed systems for the transient and the stationary regimes, but requires a large volume of calculations and using of computer-aided design.

According to these considerations, the aim of this paper is to elaborate the algorithms for the optimal synthesis of PID controllers to the various models of objects with inertia, astatism and time delay with known parameters based on the maximum stability degree criterion that would satisfy the following goals:

- to be easy to implement;
- to require a low volume of calculation,
- do not impose restrictions on the complexity of the control objects' models;
- to provide good performance of the designed systems;
- to be able to impose the desired performance;
- to be used both for synthesis of the conventional automatic systems and for synthesis of the automatic systems with auto tuning and the adaptive systems.

II. SYNTHESIS ALGORITHMS OF CONTROLLERS

The elaborated method for synthesis of PID controllers for the automatic control systems with maximum stability degree is applied to the models of objects that can be presented with the following transfer function [4, 5]

$$H_F(s) = \frac{k\exp(-\tau s)}{s^{\upsilon}\prod_{i=1}^{z}(T_i s+1)} = \frac{k\exp(-\tau s)}{a_0 s^r + a_1 s^{r-1} + ... + a_{r-1}s + a_r}, \quad (1)$$

where k is the transfer coefficient of the control object; $T_1, T_2, ..., T_z$ - time constants; τ - time delay; υ - astatism degree; a_i - transfer function coefficients of the model of control object; $r = z + \upsilon$ - degree of the object's model.

The transfer function of the PID controller is presented in the following form

$$H_{PID}(s) = k_p + \frac{1}{T_i s} + k_d s = k_p + \frac{k_i}{s} + k_d s\,, \quad (2)$$

where k_p, k_i, k_d are the tuning parameters of the controller; $k_i = 1/T_i$.

The control algorithm is selected from recitals that the number of tuning parameters m in the control law meets the following condition $m \le (n-1)$, where n is the degree of characteristic equation of the designed system.

Next, is determined the transfer function of the closed loop system with the control object's model (1) and the selected controller, whence the characteristic equation is obtained

$$\begin{aligned} A(p,b_j) &= a_0 p^n + \sum_{i=1}^{n} a_i p^{n-i} + ke^{-\tau p}\sum_{j=1}^{m} b_j p^{m-j} = \\ &= a_0 p^n + a_1 p^{n-1} + ... + (a_{r-1} + \exp(-\tau p)kb_{m-2})p^2 + \\ &\quad + (a_r + \exp(-\tau p)kb_{m-1})p + \exp(-\tau p)kb_m = \\ &= d_0 p^n + d_1 p^{n-1} + d_2 p^{n-2} + ... + d_{n-1}p + d_n = 0, \end{aligned} \quad (3)$$

where n is the degree of the designed automatic control system; b_j ($j = (1,...,m)$) - dynamic tuning parameters of the selected control law (for PID controllers: k_p, k_i, k_d); m - number of tuning parameters of control law; $d_0 = a_0$; $d_1 = a_1$; ...; $d_{n-1} = \; = a_r + \exp(-\tau p)kb_{m-1}$; $d_n = \exp(-\tau p)kb_m$.

The imposing to the designed control system of the maximum stability degree is provided by the following statement [6, 7]: *the maximum possible stability degree of the automatic control system can be achieved when the real parts of all roots p_i of the characteristic equation are equal.*

Proceeding from the statement above, it is introduced the notion of the maximum stability degree J and using substitution $p_i = -J$, $i = (1,...,n)$, the characteristic equation of the automatic control system is transcribed by decomposing them into n linear factors

$$\begin{aligned} A(p) &= a_0(p+J)^n = a_0(c_n p^n + c_{n-1}Jp^{n-1} + ... \\ &\quad ... + c_2 J^{n-2}p^2 + c_1 J^{n-1}p + c_0 J^n) = \\ &= q_0 p^n + q_1 p^{n-1} + q_2 p^{n-2} + ... + q_{n-1}p + q_n = 0, \end{aligned} \quad (4)$$

where $q_i = f_i(a_0, c_i, J)$, $i = (0,...,n)$.

Expressions (3) and (4) are equivalent, as represent characteristic equations of the same automatic control system. From this reason, the coefficients beside of the same order variables of these equations are equal to each other.

As can be seen, the coefficients of characteristic equation (3), with indexes $i = (n,(n-1),...,(n-(m-1)))$, include the tuning parameters b_j $(j = 1,...,m)$ of the selected controller. Therefore, equaling of the respective coefficients of (3) with the coefficients that have the same index in (4)

$$\begin{aligned} d_i(k, a_i, b_j) &= q_i(a_0, J), \\ i &= (n,(n-1),...,(n-(m-1))), \end{aligned} \quad (5)$$

after some transformations are obtained expressions for calculating the tuning parameters b_j of the chosen control algorithm

$$b_j = f_j(k, a_0, a_i, J), \quad j = (1,...,m). \quad (6)$$

Further, equaling the coefficients of the characteristic equations (3) and (4) with the next index $i = (n-m)$

$$d_i(a_i) = q_i(a_0, J, \omega_i), \; i = (n-m), \quad (7)$$

after some transformations is obtained the algebraic expression for determining the maximal stability degree J of the designed system

$$J = f(a_0, a_i). \quad (8)$$

Mention, that the coefficients of the characteristic equations (3) and (4), with the indexes $i = (n-m)$, which throught their equaling give the expression for the calculation of the maximum stability degree J, are in addition to the variables of the order that is equal to the number of tuning parameters m in the selected control law.

From the characteristic equation (4) the expression

$$\begin{aligned} (p+J)^n &= c_n p^n + c_{n-1}Jp^{n-1} + ... \\ &\quad ... + c_2 J^{n-2}p^2 + c_1 J^{n-1}p + c_0 J^n \end{aligned} \quad (9)$$

represents Newton's binomial, which coefficients are calculated as follows [8]

$$c_i = \frac{n(n-1)...(n-i+1)}{1\cdot 2...i} = \frac{\prod_{l=1}^{i}(n-l+1)}{i!}, \qquad (10)$$

where $c_0 = 1$; $i = (1,...,n)$.

Using the expression for the calculation of binomial coefficients (10), after some transformations the algorithms for synthesis of the P, PD, PI and PID controllers to the control object's models (1) with inertia of arbitrary degree *r*, astatism and time delay have been developed for automatic control systems with maximum stability degree, which are presented below.

- For P controller

$$J = \sqrt[r-1]{\frac{a_{r-1}}{ra_0}},$$
$$k_p = \frac{\exp(-\tau J)}{k}[a_0 J^r - a_r]. \qquad (11)$$

- For PD controller

$$J = \sqrt[r-2]{\frac{2a_{r-2}}{(r^2-r)a_0}},$$
$$k_p = \frac{\exp(-\tau J)}{k}[a_0 J^r - a_r], \qquad (12)$$
$$k_d = \frac{\exp(-\tau J)}{k}[ra_0 J^{r-1} - a_{r-1}].$$

- For PI controller

$$J = \sqrt[r-1]{\frac{2a_{r-1}}{(r^2+r)a_0}},$$
$$k_p = \frac{\exp(-\tau J)}{k}[(r+1)a_0 J^r - a_r], \qquad (13)$$
$$k_i = \frac{\exp(-\tau J)}{k}[a_0 J^{r+1}].$$

- For PID controller

$$J = \sqrt[r-2]{\frac{6a_{r-2}}{(r^3-r)a_0}},$$
$$k_p = \frac{\exp(-\tau J)}{k}[(r+1)a_0 J^r - a_r],$$
$$k_i = \frac{\exp(-\tau J)}{k}[a_0 J^{r+1}], \qquad (14)$$
$$k_d = \frac{\exp(-\tau J)}{k}[0,5a_0(r^2+r)J^{r-1} - a_{r-1}].$$

From (11)-(14) it is observed that the dynamic tuning parameters of controllers depend on the known object's parameters and the maximum stability degree J of the system. If, however, J is considered as a free parameter, then in accordance with the expression [4, 6]

$$t_s \approx \frac{\ln(1/\varepsilon_{st})}{J} \qquad (15)$$

may be to impose or to optimize the settling time of the designed automatic system. In (15) the ε_{st} is the stationary system error.

III. CASE STUDIES AND COMPUTER SIMULATION

To argue the applicability, efficacy and quality of the elaborated algorithms for synthesis of controllers for the automatic control system with maximum stability degree are presented several case studies and practical applications. To estimate the performance of automatic control system designed by the elaborated algorithms (11)-(14) will be used the following methods: Ziegler-Nichols [1,2], Coon [3] and Optimization Parameters (the block of parameter optimization from the package of programs Matlab Simulink).

Example 1. Assume that the control process is characterized by the object's model with third order inertia and time delay

$$H_F(s) = \frac{k\,e^{-\tau s}}{(T_1 s+1)(T_2 s+1)(T_3 s+1)} = \frac{k\,e^{-\tau s}}{a_0 s^3 + a_1 s^2 + a_2 s + a_3} \qquad (16)$$

where k is the object transfer coefficient; T_1, T_2, T_3 - time constants; τ - time delay; a_i - transfer function coefficients of the object's model. For example stable $k = 5$; $\tau = 1s$ and $a_0 = 144; a_1 = 108$; $a_2 = 20; a_3 = 1$.

It is necessary to synthesize the PI and PID controllers to the object's model (16). The results of synthesis of the controllers are shown in the Table I.

TABLE I. CONTROLLERS' SYNTHESIS TO THE OBJECT'S MODEL (16)

No.	The synthesis method	Type of controllers				
		PI		PID		
		k_p	k_i	k_p	k_i	k_d
1	Maximum Stability Degree	J=0,152		J=0,1875		
		0,176	0.0132	0,464	0,0295	1,72
2	Ziegler-Nichols	0,74	0,057	1,234	0,076	2,2
3	Coon	0,1	0,0095	0,2	0,0144	0,701
4	Parametric Optimization	0,25	0,015	0,37	0,0244	1,1

The transient processes of the designed control system are presented in the Figure 1. The numbering of the curves corresponds to the numbering of the methods presented in the Table I.

According to the simulation results were determined the performance of the control system ($\varepsilon_{st} = \pm 5\%$), shown in the Table II.

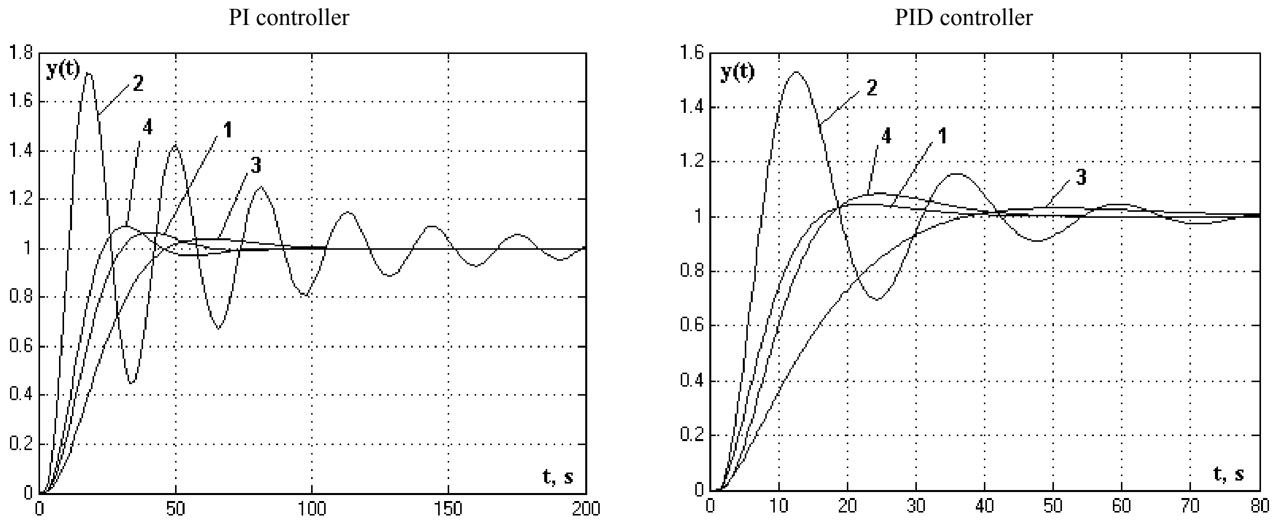

Fig. 1. The transient processes of automatic control system with the object's model (16).

TABLE II. PERFORMANCE OF THE CONTROL SYSTEM WITH OBJECT (16)

No.	The synthesis method	Type of contro-llers	System performance				
			t_r, s	t_s, s	σ, %	λ	ψ
1	Maximal Stability Degree	PI	22	45	6,4	1	1
		PID	12	14	4	-	-
2	Ziegler-Nichols	PI	6	193	72	6	0,4
		PID	4,5	51	52	2	0,7
3	Coon	PI	32	65	5,2	1	1
		PID	30	31	3	-	-
4	Parametric Optimization	PI	16	39	9	1	1
		PID	12	33	8,3	1	1

Example 2. Whether the control technologic process is presented by the object's model with second degree inertia and astatism

$$H_F(s) = \frac{k}{s(T_1 s+1)(T_2 s+1)} = \frac{k}{a_0 s^3 + a_1 s^2 + a_2 s}, \quad (17)$$

where k is the object transfer coefficient; T_1, T_2 - time constants; a_i - transfer function coefficients of the model object. For example stable $k = 2$; $T_1 = 0{,}2\,s$; $T_2 = 0{,}5\,s$ and $a_0 = 0{,}1$; $a_1 = 0{,}7$; $a_2 = 1$.

It is necessary to synthesize the P and PD controllers to the object's model (17). The results of synthesis of the controllers are shown in the Table III.

TABLE III. CONTROLLERS' SYNTHESIS TO THE OBJECT'S MODEL (17)

No.	The synthesis method	Type of controllers		
		P	PD	
		k_p	k_p	k_d
1	Maximal Stability degree	J=1,83	J=2,33	
		0,31	0,64	0,3
2	Ziegler-Nichols	1,775	-	-
3	Coon	0,5	0,5	0,116
3	Parametric Optimization	0,42	1,72	0,958

The transient processes of the designed control system are presented in the Figure 2. The numbering of the curves corresponds to the numbering of the methods presented in the Table III.

According to the simulation results were determined the performance of the automatic control system ($\varepsilon_{st} = \pm 5\%$), shown in the Table IV.

TABLE IV. PERFORMANCE OF THE CONTROL SYSTEM WITH OBJECT (17)

No.	The synthesis method	Type of controllers	System performance				
			t_r, s	t_s, s	σ, %	λ	ψ
1	Maximal Stability Degree	P	2,5	3	-	-	-
		PD	1,2	1,4	-	-	-
2	Ziegler-Nichols	P	0,52	7,8	59	3	0,6
		PD	-	-	-	-	-
3	Coon	P	1,6	4	12	1	1
		PD	1,62	2	3	-	-
4	Parametric Optimization	P	1,8	4,3	8	1	1
		PD	0,46	1,2	8	1	1

Example 3. Assume that the control process is characterized by the object's model with second order inertia and time delay

$$H_F(s) = \frac{k\exp(-\tau s)}{(T_1 s+1)(T_2 s+1)} = \frac{k\exp(-\tau s)}{a_0 s^2 + a_1 s + a_2}, \quad (18)$$

where k is the object transfer coefficient; T_1, T_2 - time constants; τ - time delay; a_i - transfer function coefficients of the model object. For example stable $k = 4$; $T_1 = 5\,s$; $T_2 = 13\,s$; $\tau = 2$; $a_0 = 65$; $a_1 = 18$; $a_2 = 1$.

It is formulated the task to synthesize the control system compound of the object's model (18) and the PID controller that would provide the different imposed values of the settling time t_s, using the elaborated algorithms in this paper.

Using (14) and (15) were calculated the dynamic parameters of PID controller to the object's model (18) which ensure transient processes with the imposed settling time.

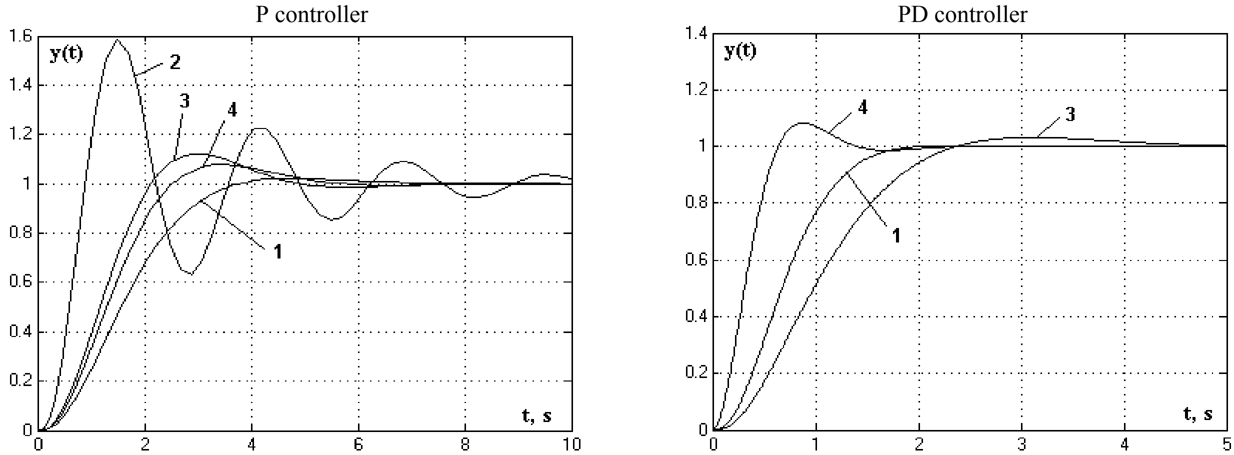

Fig. 2. The transient processes of automatic control system with the object's model (17).

The results of the synthesis are given in the Table V and in the Figure 3. It have been used the following notations: $t_{s\,imp}$ is the imposed settling time; t_s - value of settling time obtained from the transient processes of the designed system. The numbering of the curves corresponds to the numbering from the Table V.

TABLE V. CONTROLLERS' SYNTHESIS FOR DIFFERENT $t_{r\,imp}$

No.	$t_{s\,imp}$	t_s	J	k_p	k_i	k_d
1	30	32	0,1	0,272	0,0173	0,69
2	25	25	0,12	0,356	0,022	1,064
3	20	18	0,15	0,6	0,031	2
4	18	12	0,166	0,8	0,049	2,6

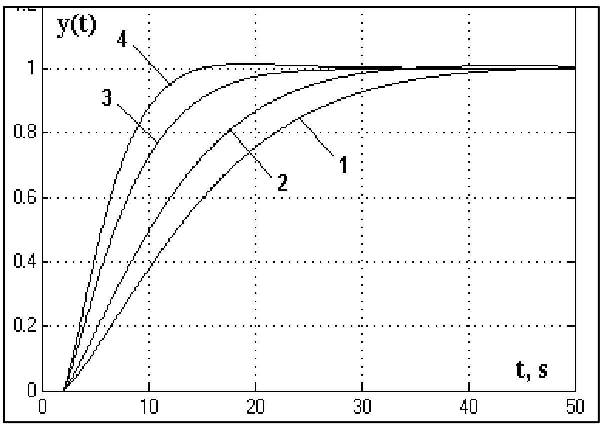

Fig. 3. The transient processes of the automatic control system for various imposed values of $t_{r\,imp}$.

It was noted that in the result of applying the elaborated algorithms the settling time t_s of the designed system is approximately equal to the imposed settling time $t_{s\,imp}$.

IV. CONCLUSIONS

In this paper have been developed the algorithms for synthesis the P, PD, PI and PID controllers to the complex objects' models, based on the maximum stability degree criterion. The elaborated algorithms satisfy the following desiderates:

- they represent a simple procedure and do not require a large volume of calculations;
- they do not impose restrictions on the complexity of the control objects and can be applied for different types of objects' models: with inertia of arbitrary degree, with inertia and astatism; with inertia and time delay, with inertia, astatism and time delay;
- in comparison with other methods of controllers' synthesis, they provide good performance to the designed automatic control systems, resulting the aperiodic transient processes;
- they allow to impose or to optimize the settling time of the designed automatic control system;
- they can be used both for synthesis of the conventional automatic systems and for synthesis of the automatic systems with auto tuning and the adaptive systems.

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