DETERMINATION ON SOME SOLUTIONS TO THE STATIONARY 2D NAVIER-STOKES EQUATION

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Consider the following system of partial differential equations:

$$\begin{cases} \frac{P_x}{\mu} + uu_x + vu_y = a(u_{xx} + u_{yy}) + F_x \\ \frac{P_y}{\mu} + uv_x + vv_y = a(v_{xx} + v_{yy}) + F_y \\ u_x + v_y = 0 \end{cases} \tag{1}$$

$$P = P(x, y); \quad u = u(x, y); \quad v = v(x, y); \quad x, y \in \mathbb{R},$$

where $P, u, v, F : D \to \mathbb{R}^2$.

The system (1) describes the process of stationary fluid flow or gas on a flat surface. The function P represents the pressure of the liquid, and functions u, v represent the flow of the liquid (gas). The constants $a > 0$ and $\mu > 0$ are determined by the parameters of the liquids (of the gas), which are viscosity and liquid's density. The function F represents the exterior forces.

Theorem. *Suppose that $u, v \in C^2(D)$ admit the bounded derivatives up to including order 2 in D.*

If $f(z)$, $z = x + iy$, is an analytical function in D, then (u, v, P), with $u = Imf$, $v = Ref$, $P = [F - 0,5(u^2 + v^2) + c]\mu$ are solutions to the system (1).

If $W(x, y)$ is a harmonic function in D, then (u, v, P), with

$$u = W_y + c_1 y + c_2, \quad v = -W_x + c_3 x + c_4,$$

$$P = [F - 0,5(u^2 + v^2) + (c_1 - c_3)W + 0,5(c_1 y^2 - c_3 x^2) + c_2 y - c_4 x + c]\mu,$$

and the arbitrary constants c, c_1, c_2, c_3, c_4 are solutions to the system (1).

In addition, various special cases were studied, and particular and exact solutions of the system (1) were found in these cases.