Analytical Algorithms for Synthesis of PID Controllers to the Complex Objects

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***Abstract*—In this paper it is proposed to extend the maximum stability degree criterion method for synthesis of the typical controllers to the model of objects with arbitrary order inertia and non-minimum phase, astatism and time delay for the automatic control systems with aperiodic response. There are designed the analytical algorithms for synthesis of the P, PI, PD and PID controllers, which represent simple algebraic expressions, that require the reduced volume of calculations and not impose the restrictions on the complexity of the control object. It was analyzed the efficiency of the developed algorithms and some study cases are presented.**

Keywords—the complex model of objects, model of objects with inertia, astatism, non-minimum phase, time delay, the typical control algorithms, the automatic control system with maximum stability degree, the synthesis methods, analytical algorithms for synthesis the controller

I. INTRODUCTION

Nowadays, control problems of the complex objects keep the significant place in the theory and practice of analysis the dynamic systems and synthesis of the automatic control algorithms. In a number of practical applications of the system theory, it is necessary to be taken into account such features of real-object's models, as input-output connections of the sub-processes, properties of minimum or non-minimum phase, astatism (with pole in the origin) and time delay. As examples of such objects can be considered balancing mechanisms, some mechatronic systems, steam generators, aircrafts, chemical reactors, hydro dynamical systems and a vast number of other systems. Solving the control problem for such systems remains actual in the automatic control theory [1-5].

These kind of objects can be described by the following transfer function

$$H_p(s) = \frac{\exp(-\tau s)\left(b_0 s^l + b_1 s^{l-1} + \dots + b_l\right)}{a_0 s^r + a_1 s^{r-1} + \dots + a_{r-1} s + a_r}, \qquad (1)$$

where $a_0, \dots, a_r$ are constant coefficients, which express the internal properties of the system and $b_0, \dots, b_l$ - express the properties of the input signal; τ is the time delay.

For synthesis the typical controllers P, PD, PI and PID can be used a lot of known tuning methods [6-11]. Analytical methods (Coon, Shedel, the method of the module, the method of symmetry etc.) are based on the approximation of the process dynamics with low order models; the graph-analytical methods (roots locus method, frequency method) provide satisfactory performance for the designed control system, but require a large volume of calculations with graphical representation; the parametrical optimization method offers good results, but requires a large volume of calculation and in some cases the calculation procedure can diverge from the optimal values. Presence of time delay and non-minimum phase properties in the model of the control object complicates the synthesis procedure of the controller, and many synthesis methods, as the experimental methods (Ziegler-Nichols, Chien-Hrones-Reswich, Offerens etc.) for example, in this case are not applicable.

One of the criteria, used to synthesize of the automatic controllers, it is the maximum stability degree criterion. The designed systems, based on this criterion, are characterized by the high speed, low overshoot and robustness at the control object's parameter variation [12-14].

In [15], based on the maximum stability degree criterion has been developed a method for tuning of the typical controllers to the models of objects with inertia and time delay. At the same time, the procedure of synthesis the controllers is substantially simplified, in case then it is operated with simple analytical expressions, that depends on the control object's parameters.

Based on these considerations, in this paper it is proposed to extend the maximum stability degree method to the model of objects with arbitrary order inertia, astatism, non-minimum phase and time delay and to elaborate the analytical algorithms for synthesis of typical controllers, which are presented in form of algebraic expressions.

II. ANALYTICAL ALGORITHMS FOR SYNTHESIS THE CONTROLLERS

It is considered that the mathematical model of the control object is described by the transfer function (1).

The controller represents the general form of the PID controller and its variation, which is characterized by the following transfer function

$$H_c(s) = k_p + k_i/s + k_d s = (k_d s^2 + k_p s + k_i)/s, \qquad (2)$$

where k_p, k_i, k_d are the tuning dynamical parameters of the controller. It is denoted with m – the number of tuning parameters in the respectively control law (P, PD, PI or PID).

The characteristic equation of the system (1), (2) is represented by the following expression:

$$A(s) = \sum_{i=0}^{n} a_i s^{n-i} + \exp(-\tau s)\left(k_d s^2 + k_p s + k_i\right)\sum_{j=0}^{l} b_j s^{l-j} =$$
$$= a_0 s^n + a_1 s^{n-1} + \dots + \qquad (3)$$