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# Constructal tree shaped networks for the distribution of electrical power

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**Abstract**

In this paper, we extend to the design of electric power distribution networks the constructal method of deducing the multiple dimensions of the network from the maximization of global system performance. Unlike earlier constructal designs of tree shaped networks, where the global objective was minimization of flow resistance and exergy destruction, in the present study, the global objective is minimization of the present worth total cost. The first half of the paper is a detailed account of all the components of the distribution system (hierarchy, voltage levels, lines, transformers) and the associated cost components that make up the global cost function. Emphasis is placed on the relations between length scales (radii, or reaches) at every voltage level and the costs of components and assemblies of components at every level and, ultimately, at the global level. It is shown that the global cost depends on multiple length scales, the load density (power consumption per unit of territory served) and the operating and economic characteristics of the voltage lines and transformers. Tradeoffs between the costs of lines and transformers exist at every voltage level, and this permits the multiple minimization of the global cost function. The optimized radii for each voltage level of the network are reported: they decrease at their own rates as the load density increases.



*Keywords:* Constructal design; Power distribution; Tree networks; Topology optimization; Economic optimization; Geometry optimization; Dendritic

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## Nomenclature

| | |
|---|---|
| $a$ | specific investment cost for line segment with cross-section $F_{0,\mathrm{ec}}$, \$/km |
| $b_0$, $b_1$, $b_2$ | coefficients of transformer total cost polynomial equation |
| $c_1$, $c_2$ | coefficients of total cost polynomial equation for given voltage-line system |
| $\cos\varphi$ | power factor |
| $c_\mathrm{p}$ | average investment cost of power system, \$/kW |
| $c_\mathrm{w}$ | incremental cost of electric energy, \$/kW h |
| $C_\mathrm{B}$ | book value at end of study period, \$ |
| $C_\mathrm{I}$ | investment cost, \$ |
| $C_\mathrm{Il}$ | line investment cost for elementary network, \$ |
| $C_\mathrm{It}$ | transformer investment cost, \$ |
| $C_\mathrm{M}$ | maintenance cost, \$ |
| $C_\mathrm{o}$ | operating cost, \$ |
| $C'_\mathrm{pw}$ | cost of variable demand and energy losses over study period expressed per unit of demand loss, \$/kW |
| $C'_\mathrm{PWT}$ | cost of fixed demand and energy losses over study period expressed per unit of demand loss, \$/kW |
| CPW | total cost of demand and energy losses during study period, \$ |
| CPW′ | load related demand and energy losses cost, \$ |
| CPW″ | zero load demand and energy losses cost, \$ |
| $C_\mathrm{s}$ | salvage value, \$ |
| $d$ | discount rate for future costs |
| EHV | extra high voltage |
| $E_\Sigma$ | coefficient integrating some cost items in discounted total cost equation |
| $F$ | conductor size (cross-section) for line segment, mm$^2$ |
| $F_{0,\mathrm{ec}}$ | conductor economic size for $I_0$ line load, mm$^2$ |
| $g$ | coefficient in total cost equation for given voltage-line system |
| $i$ | ordinal index for line segment groups |
| $I_\mathrm{c}$ | network node current load, A |
| $I_i$ | current load for $i$th group of line segments, A |
| $I_0$ | individual consumer current load, A |
| $j_\mathrm{ec}$ | economic current density, A/mm$^2$ |
| $k_\mathrm{p}$ | growth factor for variable demand loss cost over study period |
| $k_\mathrm{w}$ | growth factor for variable energy loss cost over study period |
| $K_{\mathrm{load},t}$ | load growth factor at year $t$ |
| $k_\mathrm{Il}$ | variable cost of line per unit of cross-sectional area and length, \$/km mm$^2$ |
| $L$ | line segment length, km |
| $L_0$ | half of distance between two individual consumers, km |
| $L_\Sigma$ | total length of line routing, km |
| $n$ | number of segment groups for elementary network |
| $N_\mathrm{l}$ | number of line segments for elementary network |

$N_t$ number of transformers
PDS power distribution system
$P_0$ individual consumer power load, kW
$q$ total cost of no load demand and energy losses over study period per line length unit, \$/km
$r$ network economic reach, km
$R$ resistance of line/transformer, Ω
$S$ total area served, $km^2$
$S_t$ transformer loading, kV A
$S_{tn}$ transformer capacity, kV A
$t$ index for one year in lifetime of equipment
$T$ planning period, years
$T_{life}$ useful life of equipment, years
$\overline{T}$ discounted planning period, years
TCD discounted global cost of distribution system over study period, \$
$TCD_l$ discounted total cost for elementary network, \$
$TCD_L$ total cost for given voltage-line system, \$
$TCD_t$ total transformer cost, \$
$TCD_T$ total cost of transformers supplying line system, \$
$U$ nominal voltage, kV

*Greeks*
$\alpha_B$ discounted book value coefficient
$\alpha_M$ annual maintenance charge rate, given as a decimal
$\delta$ annual load growth rate, %/year
$\Delta P$, $\Delta p$ demand loss, kW
$\Delta P'$ load related demand loss, kW
$\Delta P''$ zero load power loss, kW
$\theta$ reference year for discounting future costs
$\rho$ specified conductor resistivity, $\Omega\,mm^2/km$
$\sigma$ surface load density, $kW/km^2$
$\tau$ losses duration, h/year

*Subscripts*
B book value
dis discounted values
ec economic
f fixed
HV high voltage
I investment
l power line
L line system
LV low voltage

| | |
|---|---|
| M | maintenance |
| MV | medium voltage |
| t | transformer |
| T | transformers supplying line system |
| $v$ | variable, voltage-line system |

*Superscripts*

| | |
|---|---|
| $(\ )'$ | load related |
| $(\ )''$ | zero load related |

## 1. Introduction

During the last 100 years, the use of electrical power has penetrated every sector of human life. Electrical machines, installations and appliances define the way in which we live and work. Most people, who are unaware of the generation and distribution of electrical power, take a miracle for granted—the power for lighting, cooking, heating, working and other basic needs—they all come from a wall socket.

Behind the sockets, and behind the wiring in the walls, thrives a world of engineers, engineered networks and systems. What is the structure of these large and complex flow systems, and what makes a structure better? What are the basic concepts, guiding principles and design solutions for modern distribution networks? These are the questions that we address in this paper. We consider them important and timely, not only for their immediate relevance to the design of flow systems in general (e.g. heat, fluids, goods, capital, information, urban traffic), but also for the constructal theory implication that the same principles govern the generation of geometric structure in natural flow systems [1].

In this paper, we extend the constructal design method to the optimization of distribution networks for electric power. We will show that the dimensions of the network result from the maximization of global performance. Unlike in the tree flows optimized in the past [1], where the global objective was minimization of flow resistance and exergy destruction, in this study, we focus on minimization of the total cost of the distribution system. For simplicity, we assume that the consumers are distributed uniformly over the territory and that they are supplied by a single power source. Various means of transport (lines, transformers, voltage levels) connect the consumers to the source. We will show the tradeoffs that exist between some of the cost components and that all the dimensions of the network can be derived in the pursuit of minimal global cost.

## 2. The hierarchical structure of distribution systems

Distribution systems are an integral part of electrical power systems. Power systems are highly complex and cover immense territories. They change in time. They evolve and grow in order to meet increasing demands in electrical power [2–4]. They connect neighborhoods, cities, counties, countries and continents.