Comm. Math. Sci.
Vol. 2, No. 1, pp. 1–18

RECOVERY-BASED ERROR ESTIMATION AND ADAPTIVE SOLUTION OF ELLIPTIC VARIATIONAL INEQUALITIES OF THE SECOND KIND*

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Abstract. In this paper, we present and analyze gradient recovery type a posteriori error estimates for the finite element approximation of elliptic variational inequalities of the second kind. Both reliability and efficiency of the estimates are addressed. Some numerical results are reported, showing the effectiveness of the error estimates in adaptive solution of elliptic variational inequalities of the second kind.

1. Introduction

A posteriori error analysis has become an important tool for reliability assessment and efficiency improvement of the finite element method in solving both linear and nonlinear problems. An important class of a posteriori error estimates is based on local or global averaging of the gradient, e.g. in the form of Zienkiewicz-Zhu gradient recovery technique, [19, 20, 21]. It is known that in the case of structured grids and higher regularity solutions, such estimators are both efficient and reliable. Some work has been done for unstructured meshes as well, e.g. [16, 18]. A systematic study of various averaging techniques for a posteriori error estimation can be found in [2, 8].

The papers mentioned above are on a posteriori error estimation for solving elliptic boundary value problems of partial differential equations. For numerical solutions of variational inequalities, a few papers can be found for their a posteriori error analysis. E.g., residual type error estimators were obtained for an elliptic obstacle problem in [10] and for an elliptic variational inequality of the second kind in [5], and gradient recovery type error estimates for an elliptic obstacle problem have been shown recently in [3, 17].

In this paper, we derive and study a posteriori error estimates of gradient recovery type for finite element solutions of elliptic variational inequalities of the second kind. The paper is organized as follows. In Section 2 we introduce the model problem. Section 3 contains the finite element method setting. In Section 4 we derive a posteriori error estimates based on both global and local averaging techniques. Section 5 is devoted to an analysis of the efficiency of the estimators. In Section 6 we report some numerical results. We restrict ourselves in this paper to linear elements since they are popularly used in solving variational inequalities due to a lack of higher solution regularity. Also, our discussion focuses on one particular gradient recovery technique. The discussion and results presented here can be extended to other elements and other averaging techniques as studied in [2, 8].

We now list some notations used in the paper. Let Ω be a bounded domain in $\mathbb{R}^d$, $d \geq 1$, with Lipschitz boundary $\Gamma = \partial\Omega$. Let Γ_D be a closed subset of Γ with $\text{meas}(\Gamma_D) > 0$ and Γ_C be the remaining part. For any open subset ω of Ω with Lipschitz boundary $\partial\omega$, we denote by $H^m(\omega)$, $L^2(\omega)$ and $L^2(\partial\omega)$ the usual Sobolev

*Received: December 4, 2003; accepted (in revised version): January 15, 2004. Communicated by Shi Jin.
This work was supported by NSF under grant DMS-0106781.
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