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Analytic description of teeth profile of gears with rollers a[b] and justification of precessional gear parameters selection a

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Abstract. The engineering complex study of the triad "gear-technology-transmission" has permitted to elaborate a new type of mechanical transmission – planetary precessional transmissions with multicouple gear. In this paper, the authors present the mathematic model of the multicouple gear. A computer program for doing this it is also elaborated. It is shown the calculus modalities and some teeth profile diagrams.

Key words: precessional transmission, multicouple gear, gear technology.

1. Introduction

Gearings are considered the most sophisticated components of machines. Machine reliability depends very much on the gearing mechanical transmission operation, in general. The quality indices of traditional gears were increased largely by changing involute gearings, and by creating new gearings, such as Novikov-Hlebanija [1], Symark [2], etc.

In the field of planetary transmission it was considered properly to follow the way of developing new types with increased performances. Scientific analysts consider that in the field of technical sciences worldwide an essentially new type of mechanical transmission is being invented every 20-25 years. Thus, the German engineer L. Braren developed the cycloid planetary transmission "CYCLO" in 1923 [3]. The Russian engineer A. Moskvitin invented the harmonic friction transmission in 1944 [4] and in 1959 the American engineer C.W. Musser developed the harmonic gear transmission [4].

In the late 70ies a new type of mechanical transmission has been developed at the Polytechnic Institute of Chisinau (now the Technical University of Moldova). The

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new type of mechanical transmission entered into international terminology circuit as planetary precessional transmission (PPT). The first patent was issued under this name in 1983 [5,6]. Planetary precessional transmission differs from the classical one by the new principle of motion and load transformation and transmission, i.e. by using sphere-spatial motion of the satellite and variable convex – concave profile. Due to these innovative features gearing multiplicity in planetary precessional transmission reaches 100% (in classical transmissions - 3-7%) which provides increased bearing capacity, reduced dimensions and weight, extended kinematical range ± 10 ... ± 3599 (in harmonic transmissions 79 ... 300), high kinematical accuracy, etc. The research team involved in research on precessional planetary transmissions published over 800 scientific articles, obtained about 170 patents, implemented about 20 practical achievements in the field of fine mechanics and specialized technological equipment, in robotic complexes for the exploration of ferro - manganese concretions from the World Ocean bottom (USSR concept), in spaceflight technique, etc.

Know-how in the elaboration of multicouple precessional gear, manufacturing technology and control methods, and a range of precessional transmission diagrams belong to the research team from the Technical University of Moldova.

The specific character of sphere-spatial (precessional motions of the precessional transmissions pinion make impossible the utilisation of classical involute teeth profiles. This fact requires the elaboration of new profiles adequate to the sphere-spatial motion of pinion, which would ensure high performances to the precessional transmission.

In the complexity of problem "*gear-synthesis-profile study-manufacturing*" the elaboration of efficient methods of teeth manufacturing which ensures a maximum productivity and reduced cost while satisfying the requirements related to the gear with precessional motion plays an important role. To solve this problem the following has been done:

- we elaborated the mathematical model of teeth generation which shows the interaction of teeth in precessional gear;
- we investigated the kinematics of the mechanism of method realisation for teeth generation;
- we determined the tool path of motion and the family envelope of the generating surface by using the computer;
- we elaborated and manufactured from metal milling and tooth grinding tools, inclusively their longitudinal modification.

2. Analytical description of teeth profile

Teeth profiles have an important role in the efficient transformation of motion in the precessional transmissions that operate as multiplier. Multiple precessional gear theory, previously developed, did not take into consideration the influence of the diagram error of the linking mechanism in the processing device for gear wheel on the teeth profile. Functioning under the multiplication regime, these errors have major influence, which can lead to instant blocking of gear and to power losses. With this purpose, a thorough analysis was conducted on the motion development mechanism under multiplication, and on the teeth profile error generating source. On the basis of fundamental theory of multiple precessional gear, previously developed, a new gear with modified teeth profile and the technology for its industrial manufacturing was patented [7].

Cinematically, the link between the semi product and the tool, in which one of them (the tool) makes spherical-spatial motion being, at the same time, limited from rotating around the axis of the main shaft of the teething machine tool, is similar to the „satellite-driven shaft” link from the precessional planetary transmission of the K-H-V type. The kinematical link between the tool and the stationary part of the device represents a Hooke articulation that generates the variability of transfer function in the kinematical link „*tool-semi product*”. This variation will influence the teeth profile. Thus, the connection of tool with the housing registers a certain diagram error $\Delta\psi_3$ (to understand the deviation of the semi product angle of rotation ψ_3 from the angle of rotation of the semi product itself ψ_3^m at its uniform rotation):

$$u_{31}^m = -\frac{z_2 - z_3}{z_3}; \Delta\psi_3 = \psi_3 - u_{31}^m = \frac{z_2}{z_3}\left(\psi - arctg(\cos\theta \cdot tg\psi)\right). \qquad (1)$$

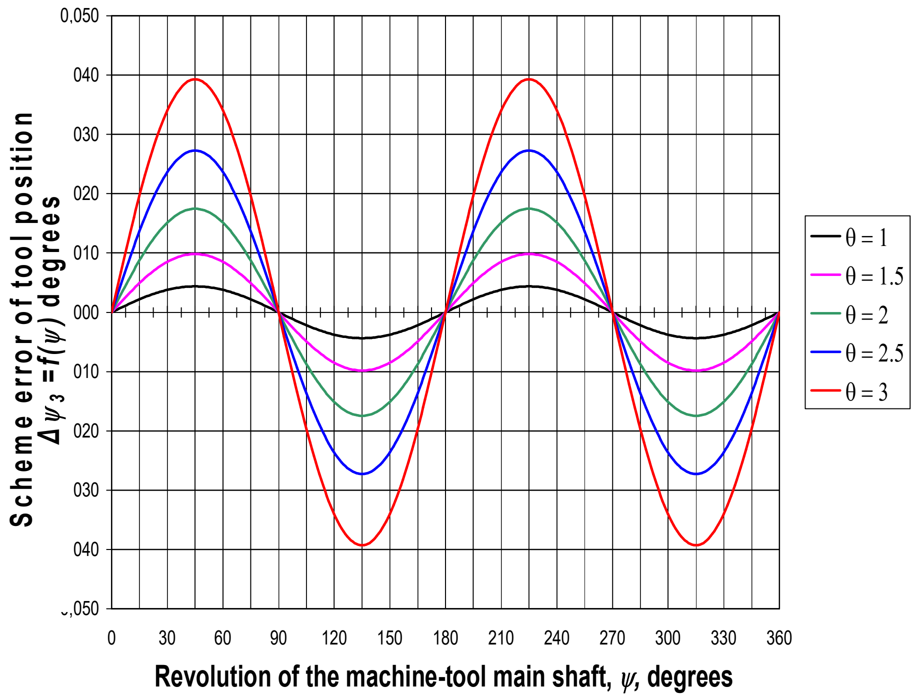


Fig. 1. Dependence of the scheme error of tool position $\Delta\psi_3$ at a revolution of the machine-tool main shaft ψ.

Fig. 1 shows the dependence of the tool position diagram error $\Delta\psi_3$ at a revolution of the machine tool main shaft ψ. This error is transmitted to the tool that shapes the teeth profile with the same error. To ensure continuity of the transfer function and to improve the performances of precessional transmission under multiplication it is necessary to modify teeth profile with the diagram error value $\Delta\psi_3$ by communicating supplementary motion to the tool. In this case the momentary transmission ratio of the manufactured gear will be constant. Usually, in theoretical mechanics the position of the body making spherical-spatial motion is described by Euler angles. The mobile coordinate system $OX_1Y_1Z_1$ is connected rigidly with the satellite wheel, which origin coincides with the centre of precession *0* (fig. 2) and performs spherical-spatial motion together with the satellite wheel relative to the motionless coordinate system *OXYZ*.

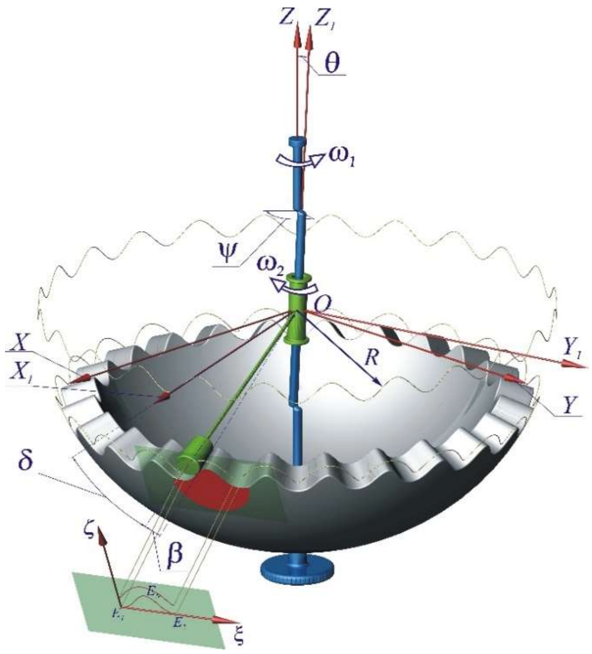


Fig. 2. Tooth profile in normal section.

The elaboration of the mathematic model of the modified teeth profile is based integrally on the mathematic model of teeth profile, previously developed by the authors. With this purpose it is necessary to present the detailed description of teeth profile without modification and, then, to present of the description of modified profile peculiarities.

2.1. Description of teeth profile designed on sphere

An arbitrary point ***D*** of the tool axis describes a trajectory relative to the fixed system according to the equations:

$$\begin{aligned}
X_D^m &= -\sin\delta \sin\left[Y_C^m \sin\theta + Z_C^m\left(1-\cos\theta\right)\cos\psi\right];\\
Y_D^m &= -Y_C^m \cos\delta + Z_C^m \sin\delta\left[\cos^2\psi + \cos\theta \sin^2\psi\right];\\
Z_D^m &= -Y_C^m \sin\delta\left(\cos^2\psi + \cos\theta\sin^2\psi\right) - Z_C^m \cos\delta.
\end{aligned} \tag{2}$$

Index m means „*modified*".

The motion of point D^m compared to the movable system connected rigidly to the semi product is described by formulas:

$$\begin{aligned}
X_{1D}^m &= X_D^m \cos\frac{\psi}{Z_1} - Y_D^m \sin\frac{\psi}{Z_1};\\
Y_{1D}^m &= X_D^m \sin\frac{\psi}{Z_1} + Y_D^m \cos\frac{\psi}{Z_1};\\
Z_{1D}^m &= Z_D^m.
\end{aligned} \tag{3}$$

The projections of point D^m velocities is expressed by formulas:

$$\begin{aligned}
\dot{X}_D^m &= -\sin\delta\cos\psi\left[Y_C^m \sin\theta + Z_C^m\left(1-\cos\theta\right)\cos\psi\right]\dot{\psi} -\\
&-\sin\delta\sin\psi\left[\dot{Y}_C^m \sin\theta + \dot{Z}_C^m\left(1-\cos\theta\right)\cos\psi - Z_C^m\left(1-\cos\theta\right)\sin\psi\cdot\dot{\psi}\right];\\
\dot{Y}_D^m &= -\dot{Y}_C^m\cos\delta + \dot{Z}_C^m \sin\delta\left[\cos^2\psi + \cos\theta\sin^2\psi\right] +\\
&+Z_C^m \sin\delta\left[-2\cos\psi\sin\psi + 2\cos\theta\sin\psi\cos\psi\right]\dot{\psi};
\end{aligned} \tag{4}$$

$$\begin{aligned}
\dot{X}_{1D}^m &= \dot{X}_D^m \cos\frac{\psi}{Z_1} - \frac{\dot{\psi}}{Z_1} X_D^m \sin\frac{\psi}{Z_1} - \dot{Y}_D^m \sin\frac{\psi}{Z_1} - \frac{\dot{\psi}}{Z_1} Y_D^m \cos\frac{\psi}{Z_1};\\
\dot{Y}_{1D}^m &= \dot{X}_D^m \sin\frac{\psi}{Z_1} + \frac{\dot{\psi}}{Z_1} X_D^m \cos\frac{\psi}{Z_1} + \dot{Y}_D^m \cos\frac{\psi}{Z_1} - \frac{\dot{\psi}}{Z_1} Y_D^m \sin\frac{\psi}{Z_1}.
\end{aligned} \tag{5}$$

The coordinates of point E^m on the sphere is calculated by formulas:

$$\begin{aligned}
X_{1E}^m &= k_2^m Z_{1E}^m + d_2^m;\\
Y_{1E}^m &= k_1^m Z_{1E}^m - d_1^m;\\
Z_{1E}^m &= \frac{(k_1^m d_1^m - k_2^m d_2^m) - \sqrt{(k_1^m d_1^m - k_2^m d_2^m)^2 + (k_1^{m2} + k_2^{m2} + 1)\cdot(R_D^2 - d_1^{m2} - d_2^{m2})}}{k_1^{m2} + k_2^{m2} + 1},
\end{aligned} \tag{6}$$

where:

$$k_1^m = \frac{X_{1D}^m\left(X_{1D}^m \dot{X}_{1D}^m + Y_{1D}^m \dot{Y}_{1D}^m\right) + Z_{1D}^{m\,2}\dot{X}_{1D}^m}{Z_{1D}^m\left(X_{1D}^m \dot{Y}_{1D}^m - Y_{1D}^m \dot{X}_{1D}^m\right)};\quad k_2^m = -\frac{\left(k_1^m Y_{1D}^m + Z_{1D}^m\right)}{X_{1D}^m};$$

$$d_1^m = \frac{R_D^2 \cos\beta\, \dot{X}_{1D}^m}{\left(X_{1D}^m \dot{Y}_{1D}^m - \dot{X}_{1D}^m Y_{1D}^m\right)};\quad d_2^m = \frac{\left(R_D^2 \cos\beta + d_1^m Y_{1D}^m\right)}{X_{1D}^m}.$$

According to the obtained analytical relations a soft for the calculation and generation of teeth was developed in CATIA V5R7 modelling system that allowed obtaining the modified trajectories of points $E^m{}_e$ and $E^m{}_i$ on the spherical front surfaces, both exterior and interior ones, by which the teeth surface was generated (fig. 3).

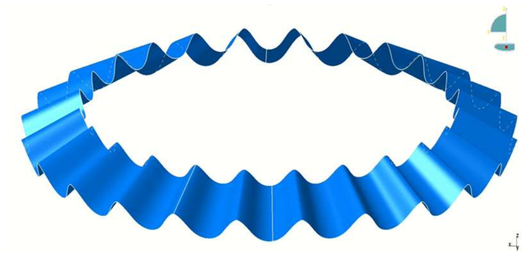


Fig. 3. Teeth generating surface.

2.2. Description of modified teeth profile projected on a transversal surface

Projection of point E^m on the tooth transversal plane has the following coordinates:

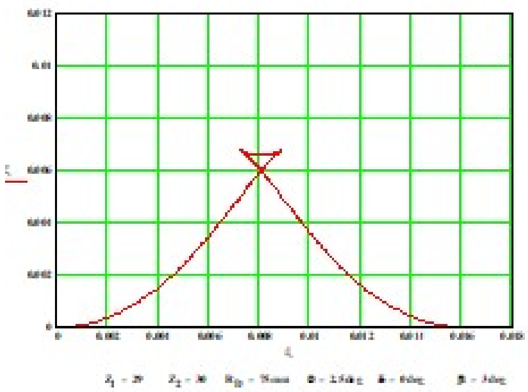
$$X_E^{\prime\prime m} = \varepsilon^m \cdot X_{1E}^m, \quad Y_E^{\prime\prime m} = \varepsilon^m \cdot Y_{1E}^{\prime\prime m}, \quad Z_E^{\prime\prime m} = \varepsilon^m \cdot Z_{1E}^{\prime\prime m}, \tag{7}$$

where $\varepsilon^m = -\dfrac{D}{AX_{1E}^m + BY_{1E}^m + CZ_{1E}^m}$.

The modified teeth profile in plane is described by the equations:

$$\begin{aligned} \xi^m &= X_E^{\prime\prime m} \cos\frac{\pi}{Z_1} + \left[R_D \cos(\delta+\theta+\beta) + Y_E^{\prime\prime m}\right] \sin\frac{\pi}{Z_1}; \\ \zeta^m &= X_E^{\prime\prime m} \sin\gamma \sin\frac{\pi}{Z_1} - \left[R_D \cos(\delta+\theta+\beta) + Y_E^{\prime\prime m}\right] \sin\gamma \cos\frac{\pi}{Z_1} + \\ &+ \left[R_D \sin(\delta+\theta+\beta) + Z_E^{\prime\prime m}\right] \cos\gamma. \end{aligned} \tag{8}$$

A wide range of modified teeth profiles with different geometrical parameters were generated in MathCAD 2001 Professional software (fig. 4). The solid model of a gear wheel is shown in fig. 5.



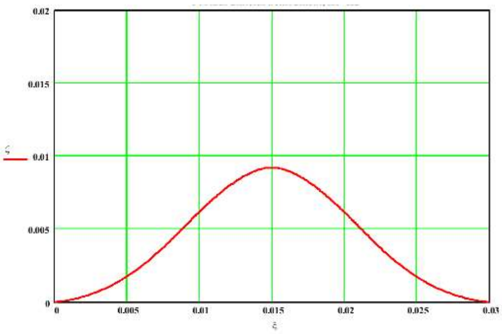


Fig. 4. Teeth profiles of weels.

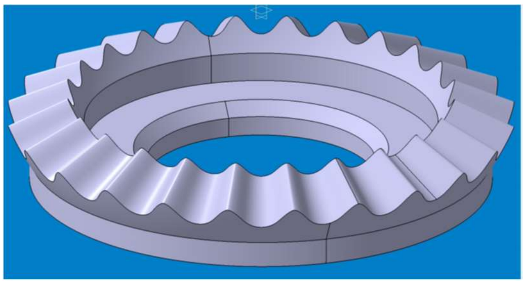
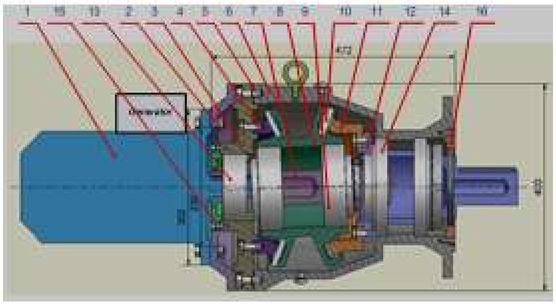


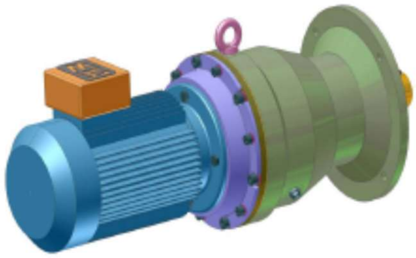
Fig. 5. Computerised model of the sun gear.

Based on the carried out research it was established that from the point of view of decreasing energy losses in gearing, in the multiplication mode of operation, the gearing angle should be $\alpha > 45^0$, and the nutation angle (the pitch angle of the crank shaft) should be – $\theta \leq 2{,}5^0$. This is dictated by the reverse principle of movement in the multipliers compared to the reducers: the axial component of the normal force in gear must be maximal to drive the crank shaft in the rotation movement through the satellite wheel.

3. Design of precessional multiplier structure

On the basis of the undertaken study, diagram 2K-H was selected for the development of planetary precessional reducer. As a result of analysis of a wide range of tooth profiles with different geometrical parameters of gear by using the mathematical modelling package MathCAD 2001 Professional, the optimum tooth profiles were selected with account of their functioning in conditions of reducing. Also, in MathCAD 2001 Professional software the calculation of geometrical parameters of precessional gear was done. The structures of planetary precessional reducer were designed in SolidWorks software. The precessional reducer is connected by flange with an electric generator, which allows obtaining a compact coaxial module. The structure from fig. 6 is proposed for various actuators.





a. *b.*

Fig. 6. Planetary precessional reducer: a – general view; b – section view.

To simulate the reducer assembly and functioning, the dynamic computerised model of the precessional reducer was developed in AutoDesk MotionInventor. The precessional reducer has reduced dimensions and mass, high lifting capacity and reduction ratio up to i = 3600) with satisfactory mechanical efficiency.

4. Conclusions

Among the characteristics of the estimated results of research in the field of new and efficient drive development we can enumerate the following:

- the elaborated precessional gears ensure: high bearing capacity; high kinematical efficiency; high kinematical accuracy; low noise level and vibrations;
- Know-how in the elaboration of the multicouple precessional gear, manufacturing technology and control methods, and a range of precessional transmission diagrams belong to research team from the Technical University of Moldova.
- Structural optimization of the precessional transmissions will allow synthesis of new diagrams of precessional transmissions with constant and variable transmission ratio and elaboration of new diagrams of precessional transmissions for specific running conditions.

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