Chapter 3
Adaptive Finite Element Solution of Variational Inequalities with Application in Contact Problems

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Summary. In this chapter, we perform a posteriori error analysis for the adaptive finite element solution of several variational inequalities, including elliptic variational inequalities of the second kind and corresponding quasistatic variational inequalities. A general framework for a posteriori error estimation is established by using duality theory in convex analysis. We then derive a posteriori error estimates of residual type and of recovery type, through particular choices of the dual variable present in the general framework. The error estimates are guaranteed to be reliable. Efficiency of the error estimators is theoretically investigated and numerically validated. Detailed derivation and analysis of the error estimates are given for a model elliptic variational inequality. Extensions of the results can be made straightforward in solving other elliptic variational inequalities of the second kind, and we present such an extension for a problem arising in frictional contact. Moreover, we use a quasistatic contact problem as an example to illustrate how to extend the a posteriori error analysis in solving time-dependent variational inequalities. Numerous numerical examples are included to illustrate the effectiveness of the a posteriori error estimates in adaptive solutions of the variational inequalities.

Key words: A posteriori error estimation, adaptive finite element solution, elliptic variational inequality, quasistatic variational inequality, frictional contact, duality, reliability, efficiency

3.1 Introduction

In this chapter, we present some theoretical and numerical results on a posteriori error estimation and adaptive finite element solution of elliptic

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