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Information model of decision making process for image recognition

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Abstract

Formalization aspects of decision making procedure under the condition of a priori uncertainty of the analyzed images are considered. The analytical schemes of decision making process are constructed. They have the same organization as pyramidal visual machines.

1 Introduction

The essence of known recognition methods consists in an estimation of a degree of similarity of entrance representation with set of the standards at a decision making stage in the assumption, that problems of initial description formation and attribute system formation are solved. Used criterions of the decision making are diverse: sufficiency of coincidence number [1], grammar and graph comparison [2,3], geometrical descriptions [4], various measures of similarity [5–7]. It is supposed, that compact set of points in the space of attributes (hypothesis of compactness [8], forming separable classes, corresponds to the standard. However, noise of measurements, interference (organized and unorganized), structural variation of the same representative of a class lead to significant extension of indicated set and, as a consequence, to overlapping of classes, therefore, to a decrease of the classification reliability. As a result the problem of decision making comes to a problem of an attribute system formation, allowing to generate such standard, which would provide conditions of compactness. Nevertheless the problem of recognition, understood in a broad sense, degenerates in a rather simple strategy, if there exists any universal (fundamental) system of transformations of any image. In works of the authors [9–13], information models of stages of the initial description formation and attributes system formation under the condition of a priori uncertainty of the image are constructed. Existence of the final set of universal operators, implementing integro-differential transformations of a kind $(dO \int_G)$ on the whole field of view G (the image definition range), "inherent" in system of the visual analyzer and forming entrance representation of any image as 16-dimensional vector of visual mass $\mathrm{M} = \{\mu_i : i = \overline{0,15}\}$ on a matrix 4×4 $(W_{4\times 4} \equiv W)$ is proved. In the present work the basic aspects of the theory of image standard synthesis and decision making under the condition of a priori uncertainty are considered.

2 Image description algebra

Let Q_i by any filter from set Q [11].

$$\{Q_0, Q_1, Q_4, Q_9; Q_2, Q_5, Q_{10}; Q_3, Q_8, Q_{15}; Q_7, Q_{12}; Q_6, Q_{11}; Q_{14}; Q_{13}\} \cong$$

$$\begin{aligned} \cong Q = \{Q^{0,0}_{xy}; Q^{1,0}_{xy}, Q^{2,0}_{xy}; Q^{3,0}_{xy}, Q^{0,1}_{xy}; Q^{0,2}_{xy}, Q^{0,3}_{xy}; Q^{1,1}_{xy}, Q^{2,2}_{xy}; Q^{3,3}_{xy},\\ Q^{1,2}_{xy}; Q^{1,3}_{xy}; Q^{1,1}_{xy}; Q^{2,3}_{xy}, Q^{3,2}_{xy}\} \cong\\ \cong \{1; v_x, v_x^2, v_x^3; v_y, v_y^2, v_y^3; v_{xy}^2, v_{xy}^4, v_{xy}^6; v_x v_y^2;\\ v_x v_y^3; v_x^2 v_y; v_x^2 v_y^3; v_x^3 v_y^2, \} \end{aligned} \tag{1}$$

where for $Q^{i,j}_{xy}$ indexes i, j is the order of the filter (transformation) in the direction of x, y accordingly; v_k^r is the differential transformation $\partial^r / \partial k^r$ in the direction k; l is the integrated transformation on area G. Let the set Q is a Q — pyramid [11]. Then the set $\{\mu_i\}$, obtained on a Q — pyramid is set of the primitive attributes of the image . As each primitive μ_i belongs to the set of real numbers and contains the information of image visual mass (as a measure) on the filter Q_i, it is allowable to put into correspondence to set (1) the set of matrices (operators) $V = \{V_i\}$, consisting of 0 and 1 and ordered by elements according to constructive definition of set (1). We shall introduce two binary operations for elements of matrices $V_i, V_j \in V$:

addition

$$[a^i_{mn}] + [a^j_{mn}] = [a^i_{mn} + a^j_{mn}] \qquad \forall n, m \tag{2}$$

multiplication

$$[a^i_{mn}] \cdot [a^j_{mn}] = [a^i_{mn} \cdot a^j_{mn}] \qquad \forall n, m \tag{3}$$

such, that for them operations of Boolean algebra are correct

$$\begin{gathered} 0+1=0, \quad 1+0=1, \quad 0+0=0, \quad 1+1=1, \\ 0 \cdot 1 = 0, \quad 1 \cdot 0 = 0, \quad 0 \cdot 0 = 0, \quad 1 \cdot 1 = 1. \end{gathered} \tag{4}$$

All operators by condition are determined in area G of the image, and as (2), (3) are ordered on elements of matrices V_i, V_j, their action is a **covering, that is direct sum**. Let V_i is a set, and each its element a^i_{mn} — single-dot subset. Then the operation (2), (3) is operations of association Y and crossing I accordingly, for which (4) is correct. Existence of upper and lower bounds follows from here. As for operations (4) laws of commutativity, associativity, idempotency and distributivity are satisfied, and the operations (2), (3) are ordered, the enumerated laws are correct and for (2), (3), i.e. for any $V_i, V_j, V_k \in V$ axioms (here and further operations of association and crossing we shall designate $< + >$, $\quad < \cdot >$ accordingly). Then triple $< V; I; Y >$ is the distributive limited structure, and since for each element of set V there is the inverce, it is a boolean algebraic structure (Boolean algebra).

Definition 1 *As an image description algebra (or simple image algebra) M a class $A(V)$ of objects $\{V_i\}$ is called, in which:*

1) *for any V_i binary operations (2), (3) with according axiomatics are determined;*

2) *V_i belongs $A(V)$ (clousereness);*

3) *zero and single elements* $e_1 = V_0 = [1]$, $e_0 = \overline{V_0} = [0]$ *are determined;*

4) *function* $M = f(V_0(\mu_0), V_1(\mu_1), \cdots, V_{15}(\mu_{15})$ *corresponds to any* M.

Let $\{V_i(\mu_i)\}$ by the set of individual objects forming the subject area J. Let the expression about the individuals is a predicate of kind $P = P(V_i, \cdots, V_k, \cdots, V_j) \in P(\{V_k\})$, $\{V_k\} \subseteq V$. Then is correct

Statement 1 *Let* $\{\mu_i : i = \overline{0,15}\}$ *is the result of decomposition of the image on a* Q *— pyramid (1). Let* $\{V_i\} = V$ *is a set of the operators satisfying to the condition* $\{V_k\} \cong \{Q_{xy}^{i,j}\}$. *Then:*

1) $P(V_i)$ *is the property of the individual* $V_i(\mu_i)$ *provided that for all* $\mu_j, (j \neq i)$, $\mu_j = 0$ *is satisfied;*

2) $P(\{V_k\})$ *is the predicate, describing a relation among elements of set* μ_k, *where* $\mu_k \neq 0 \quad \forall k$, *is defined by the order and position in series (1).*

Let $\{\mu_i\}$ by the result of decomposition of any image. Each value of μ_i contains the information about mass of a subarea of image definition range, covered with the filter $Q_i \in Q$. Then, the greafer is $\mid \mu_i \mid$ on the set $\{\mid \mu_i \mid\}$, the higher is the weight of he appropriate filter Q_i (so, operator V_i) in general decomposition (1). Therefore is correct

Statement 2 *Let* $M = \{\mid \mu_i \mid : i = \overline{0,15}\}$ *is the result of the image* M *decomposition. Then individual share of each* V_i *belonging to set* $\{V_i : i = \overline{1,15}\}$ *is defined by mass distribution* $\mid \mu_i \mid \in \{\mid \mu_i \mid\}$.

Let $m(V)_i) = \mid \mu_i \mid$ is mass of the operator V_i. Let $\{V_i\}$ is the complete set of events [11]. Then for two operators V_i, V_j, connected by operation (2) it is correct

$$m(V_i + V_j) = m(V_i) + m(V_j) - m(V_i V_j) \tag{5}$$

where $m(V_i V_j)$ is the mass of operators V_i, V_j, connected by operation (3). We shall divide equality (5) by $m(V_0) = \mu_0$. Then we shall obtain equality of probabilities $p(V_i + V_j) = p(V_i) + p(V_j) - p(V_iV_j)$, where $p(V_i, V_j)$ is probability of join $V_iV_j = V_{ij}$. From here follows that if events V_i, V_j are independent then $(V_iV_j) = p(V_i)p(V_j)$, if events are injoint then $p(V_i + V_j) = p(V_i) + p(V_j)$. Therefore is correct

Statement 3 *Let we have homogeneous* ($|\mu_i| = \mathbf{const} \quad \forall \mu_i \neq 0$) *and unambiguous correspondence between visual mass set* $\{\mu_i : i = \overline{1,15}\}$ *and set of operators* $\{V_i\} : i = \overline{1,15}$, *i.e.* $\{\mu_i\} \longleftrightarrow \{V_i\}$. *Then:*

1) condition

$$\sum_i \frac{|\mu_i|}{\mu_0} = 1 \tag{6}$$

is necessary and enough for the description of the image with the help of operation of addition (2) operators V_i, *for which* $\mu_i \neq 0$, *i.e.*

$$M \cong \sum |V_i| \tag{7}$$

2) condition

$$\frac{|\mu_i|}{\mu_0} = 1 \qquad \forall i \tag{8}$$

is necessary and enough for the description of the image with the help of operation of multiplication (3) operators V_i, *for which* $\mu_i \neq 0$, *i.e.*

$$M = \prod_i \{V_i\} \tag{9}$$

Each operator V_i on definition reveals eight connected points if a condition $\mid \mu_i \mid = \mu_0$ is fulfilled i.e. defines the binary relation between two groups of connected points. According to statm.1 each individual $V_i(\mu_i)$, where $\mid \mu_i \mid = \mu_0$ defines a predicate $\mathrm{P}(V_i)$, characterizing individual property. As $\mid \mu_i \mid = \mu_0 \quad P(V_i)$ specifies property, inherent in the image, represented by set $\{\mu_i\}$. The set of operators $\{V_i\}$ allows operation (3), locating thus some group of points (single or zero) on a matrix 4×4, belonging to a predicate $P(\{V_k\})$ on operation (3). We shall name this group of points as compactum.

Consequence 1 *A predicate on m of a variable kind*

$$P = \prod_{i=0}^{m-1} V_i = \prod_{i=1}^{m-1} V_i \tag{10}$$

(V_i is direct or inverse operators) locates m-arity compactum and is the description of set $\{V_i\}$.

From here it should be that a correct predicate $P = V_1 V_2 \dots V_{15}$ localizes 16-arity compactum which we shall name as a key or composition centre of the image.

3 Group of image algebra

We shall introduce the following concept

Definition 2 *Let $\{V_i\}$ be any set of the direct, inverse operators, belonging V. The set $\{V_i\}$ is called as complete group $P_n = (\{V_i\})$ if and only if the conditions are fulfilled*

$$\sum_i V_i = e_1; \quad \prod_i V_i = P \equiv P_n \tag{11}$$

Let P_n be any complete group of family S_n. Then, as the predicate (11) is the description of group P_n, from a condition (8) rule of complete group detection and condition of representation of the complete image group follows.

Consequence 2 *Let $\{V_i\} \in P_n$. Then $M \cong P_n$ (the ϵ-exact identity), if a condition is fulfilled*

$$\mid \mu_i \mid = \mu_0 \quad \forall i \tag{12}$$

By definition any complete group consists of arbitrary number of operators under condition of fulfilment (11). It is natural, that the unique complete group of one operator is the unit of set V, i.e. operator V_0. Besides any two operators $V_i, V_j \in V$, where $i \neq j$, will not form a complete group, since for them the first condition (11) is fulfilled. But for any two operators V_i, V_j always it is possible to find the third V_k, forming with them a complete group, as the transformations appropriate to the operators from set (1) admit operations of vector and (or) scalar multiplication [10].

Lemma 1 *Let $V_i, V_j \in V$, where $i \neq j$; $i, j \neq 0$. Then there is the third operator $V_k \in V$, where $k \neq 0$, ensuring closureness of group (V_i, V_j, V_k) concerning conditions (11).*

We shall select properties of complete group of three operators $\mathrm{P}_n = (V_i, V_j, V_k)$ taking into account the results obtained above. The description of group is a predicate $P = V_0 V_i V_j V_k \equiv V_i V_j V_k$. Then the group P_n locates 4-arity compactum. By virtue of **Concequence 2** for group condition $\mid \mu_i \mid = \mid \mu_j \mid = \mid \mu_k \mid = \mu_0$ is correct, so condition (8) is correct too. As the complete group is function of three Boolean variables, it is correct

Lemma 2 *Let (V_i, V_j, V_k) is complete group. Then the condition $\{P_{np}\} \in P(V_i, \overline{V_i}, V_j, \overline{V_j}, V_k, \overline{V_k}) = T$ (truth) is fulfilled if and only if the group contains even number of the inverse operators.*

Let $\{P_{np}\}$ is the set of complete groups of a kind (V_i, V_j, V_k). Then the sentences

$$S_1 = \bigvee_{p=1}^{4} P_{np}, \qquad S_2 = \bigwedge_{p=1}^{4} P_{np} \tag{13}$$

describe the set of situations of 4-arity compactum in area G, represented by matrix 4×4, and movement of 4-arity compactum in this area accordingly. By definition set $\{V_i\}$ is ordered on two dimensional lattice $V(x, y)$, i.e. $V = V(x, y) = V_x \times V_y$, where $V_x = \{V_m\} \cong \{\partial^i \big/ \partial x^i\}, V_y = \{V_n\} \cong \{\partial^i \big/ \partial y^i\}$ [10]. Therefore the set V is possible to consider as set of tops, to each of them the operator is put in correspondence. Then subset belonging V for which family of pairs $\mathrm{E} = (V_k, V_r)$; $V_k, V_r \in V$ is determined which we shall call as graph $G = G(V)$.

Let (V_i, V_j, V_k) be a complete group. Each operator of group by definition has the order of the appropriate transformation (1). Then the complete group may be defined as ordered group.

Definition 3 *Decomposition of any image represented by a matrix* 4×4 *on ordered set*

$$V = \left\{ \begin{pmatrix} V_2 \\ V_1 \end{pmatrix}, \begin{pmatrix} V_5 \\ V_4 \end{pmatrix}, V_3, \begin{pmatrix} V_{10} \\ V_9 \end{pmatrix}, \begin{pmatrix} V_7 \\ V_6 \end{pmatrix}, \begin{pmatrix} V_{12} \\ V_{11} \end{pmatrix}, V_8, \begin{pmatrix} V_{14} \\ V_{13} \end{pmatrix}, V_{15} \right\} \qquad (14)$$

is vector $\mathbf{M} = \{\mu(V_i)\}$ *in basis (14).*

As the set V is ordered, any graph on $V(x, y)$ is a digraph. Since any graph G on a lattice $V(x, y)$ is oriented and all operators are ordered on $V(x, y), G$ is an acyclic graph.

Lemma 3 *Let* $d(V_i, V_j)$ *is the length of a path on graph* $G \subseteq (V(x, y)$. *Then a diameter of the graph is equal to*

$$d(G) = \max_{V_i, V_j \in V} d(V_i, V_j) = 3e$$

Here e is the unit of lenght of edge of a graph on a lattice $V(x, y)$. From the set theory it is known that two crossing sets M_1, M_2 coincide on subsets, belonging M_1 and M_2, and are different on noncrossing subsets (naturally if M_1 and M_2 are not equivalen).

Definition 4 *Let* P_1, P_2 *are group. Then*

$$P_c = P_1 P_2, \qquad P_d = P_1 \overline{P}_2 + \overline{P}_1 P_2 \qquad (15)$$

are called as coincidence and difference predicates.

As operations (15) for elements P_i of some set P are determined it is admissible

Definition 5 *Let* $\{P_i\} = P$ *is set of groups satisfying (11). Let operations (14) on* P *be determined. Then algebraic structure* $A(P) = < P; P_c, P_d >$ *are called as group algebra on a lattice* $V(x, y)$.

4 Descriptive represententions formation

Vector M in basis (14) presents image M authentic and ϵ-exactly. Each operator $V_i \subseteq V(x, y)$ is provided by a structure according to the definition. Then the predicate $\mathrm{P}(\{V_i\})$ on operations (7), (9) for the image M is also authentic and ϵ-exact and can be used for construction of authentic and ϵ-exact descriptive (geometrical) descriptions of the image.

Definition 6 *Let the image M is presented in basis (14). Then its description $P(\{V_i\})$ by operations (7), (9) is called as a predicate of a pattern (simply a pattern) of image.*

We shall name a group of homogeneous (in the sense 1) elements of a matrix 4×4 forming a train in the direction of x, y, xy (or $x = -y$, i.e. $-xy$) as a segment. An elementary segment is atom (element of a matrix). Its equivalent representation is point int the middle of the appropriate element. Two or more elementary segments are called as connected if their next elements satisfy to criterion of 8-connectedness. As the elements of a segment are in equivalence relation [10], the following descriptive representations are admissible:

a) let the segment belongs to one of possible directions on a lattice $V(x, y)$. Then its representation is a line connecting all elements of a segment and taking place on the middle of appropriate cell matrices. We shall name this line the skeleton of a segment;

b) let the segment consists of the set of points and does not belong to any of possible directions on $V(x, y)$ (case of an as much as possible symmetric segment). Then its representation is a point in the centre of a segment.

On fig.1,2 the variety of skeletons variety for $\{V_i\}$ on a lattice $V(x, y)$ is presented and examples of skeleton for predicates belonging to complete are shown. As it is obvious from comparison of fig.1 and fig.3 from [13] the concept of image skeleton is equivalent to the set of tangents to an integral curve.

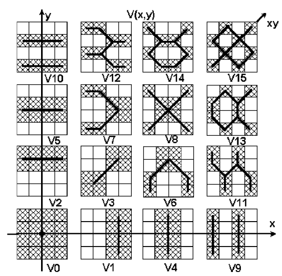

Fig.1. Variety of skeletons for set $\{V_i\}$ on a lattice $V(x,y)$

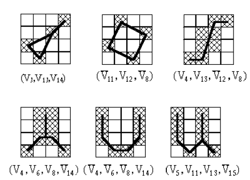

Fig.2. Examples of skeletons for groups P_n and their combinations on operation of association

5 Axiomatics of group algebra

According to the condition each operator of set $\{V_k\}$ is independent and defines k-th direction in n-dimensional space. Therefore any two operators $V_i, V_j \subseteq \{V_k\}$ are orthogonal. As for any two operators the third can be found, closing the first two in a complete group, all complete groups consist of orthogonal operators, determining three directions on a lattice $V(x, y)$.

Consequence 3 *Let $\{V_m, V_n, V_p\} \in P_n$. Then graph G simultaneously defines scalar complete group on direction $\lambda \in \{\lambda_i\}$ and vector complete group, belonging $B_3 \in \{B_i^3\}$, where λ_i is the set of directions on $V(x, y)$; $\{B_i^3\}$is the set of three-dimensional Euclidean space.*

Lemma 4 *Any pairs of noncrossing complete groups are orthogonal. By definition the set $\{V_i\}$ belongs to 15-th dimensional Killing field and defines 5-th dimensional Euclidean space [13]. Since for each direction of Euclidean spaces there corresponds complete group, then the number of such complete, noncrossing by virtue of orthogonality of directions groups should be five (fig.3). We shall name these groups basic. It is possible to select the following properties (axioms) for the set of complete groups on a lattice $V(x, y)$ (fig.3, 4).*

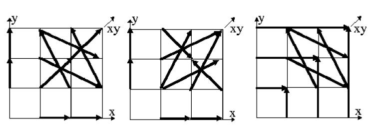

Fig.3. Family of basic complete groups S_{nb}

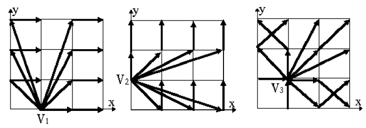

Fig.4. Examples of sets of complete groups, crossed in top V_i

Property 1. Let set of the operators $V = \{V_i : i = \overline{1,15}\}$ belongs to a two-dimesional lattice $V(x,y)$. Then:

a) the number of noncrossing complete groups is equal to 5;

b) set $\{P_{ni} : i = \overline{1,5}\}$ is complete on the set of tops of a lattice $V(x,y)$;

c) through any top of a lattice passes seven complete groups, crossing at this top.

As the set of noncrossing complete groups $\{P_{ni} : i = \overline{1,5}\}$ is complete it defines some family $S_{nb} \in S(P_n)$, which we shall name as family of basic complete groups. Besides the following identity is correct

$$\sum_{i=1}^{5} P_{ni} = e_1 \tag{16}$$

On a lattice $V(x,y)$ three orthogonal directions are determined: $x, y, xy(x = y)$. Then the set $\{P_{ni}\} \in S_{nb}$ contains a subset $\{P_{nj}\} \subset$

$\{P_{ni}\}$, each complete group of which belongs to one of directions x, y, xy. Therefore a subset $\{P_{nj}\}$ whose capacity 3 is a subset of basic complete groups of three-dimemsional Euclidean space and completely defines it (as any basis of vector space). As the groups on set $\{P_{ni}\}$ are not crossed, it is correct

Lemma 5 *Let $\{P_{ni} : i = \overline{1,5}\} \in S_{nb}$. Then:*

1. *Any subset $\{P_{nj} : j = \overline{1,3}\} \subset \{P_{ni}\}$ is a three is the dimensional basis on $V(x,y)$.*
2. *Description of a subset $\{P_{nj}\}$ of a kind*

$$P_v = \sum_{j=1}^{3} P_{nj}, \quad \overline{P}_v = \prod_{j=1}^{3} \overline{P}_{nj} \tag{17}$$

are closed $P_v + \overline{P_v} = e_1$ and are minimal with respect to operation of association (2) and crossing (3) accordingly.

6 Compactness of the pattern description

By definition for each operator and each group on $V(x,y)$ a pattern O having visual mass μ_0 as a measure exists. Let for a subset $\{V_j\} \subset V$ there be the standard O_T. Then the following measures are admissible:

1) measure of a pattern deviation O of the image M from the standard $O_T \quad P_p = O\overline{O_T} + \overline{O}O_T$;

2) measure of pattern coincidence O of the image M with $O_T \quad P_c = OO_T \equiv O_T$.

Definition 7 *Let $\{O_T\} = \tilde{M}$ is the set of standards. In this case the standard $O_T \in \tilde{M}$ and pattern $O \cong M$ are compact if only if a condition*

$$\min_{O_T \in M} \mid \mu_0 - \mu_T \mid \Rightarrow \{\min_{O_T \in M} P_p, \max_{O_T \in M} P_c\} \tag{18}$$

is fulfilled.

By definition every atom of any operator provides reception of an ϵ-exact structural element m_{ij} [12]. Then each operator V_i (or $\overline{V}_i$), if a condition (12) (so (18)) for V_i (or $\bar{V}_i$)is fuilfilled, is compact and is the standard on the set of standards $\{V_i\} \in V$.

Consequence 4 *Let for any $V_i(\bar{V}_i) \in V$ is carried out*

$$M \cong V_i \Leftrightarrow \mid \mu_i \mid = \mu_0, \quad \mu_j = 0 \forall i \neq j, \quad \forall i \neq 0 \tag{19}$$

Then $V_i(\overline{V_i})$ is the standard for M.

Similarly let for the image M a condition $M \cong \{V_i, V_j, V_k\} \in {}_{ni}$ is fulfilled. In this case is correct

Consequence 5 *Let for any complete group $P_n \in \{P_{ni}\}$ there is*

$$M \cong \begin{cases} P_n \Leftrightarrow \mid \mu_i \mid = \mu_0 & \forall V_i \in P_n, \quad \mu_j = 0 \quad \forall j \neq i \\ \overline{P_n} \Leftrightarrow \mid \mu_i \mid = m = const & \forall V_i \in P_n, \quad \mu_j = 0 \quad \forall j \neq i \end{cases} \tag{20}$$

$$(V_i, V_j, V_k) \in \begin{cases} P_n, if \ m = \mu_0 \\ \overline{P_n}, if \ m < \mu_0, \end{cases} \tag{21}$$

where ${}_n$ is defined by (11), and — $\overline{P_n}$ inversion of group P_n. Then $P_n, \overline{P_n}$, is standard for M.

Thus the sets V_i, P_{ni}, on the one hand, will form two alphabets of image description in conditions of its a priori indeterminacy, and on the other hand, are the set of standards with the appropriate descriptive represententions. Besides if each complete group on own operators triple sets three-dimemsional basis of vector space, each triple of orthogonal complete groups is 3-dimensional basis of new space, as associations of three 3-dimensional vector spaces, each of which belongs to orthogonal l directions (fig.5). Similarly if the image description algebra is determined on set $\{V_i\}$ the algebra of groups is determined on a set of groups.

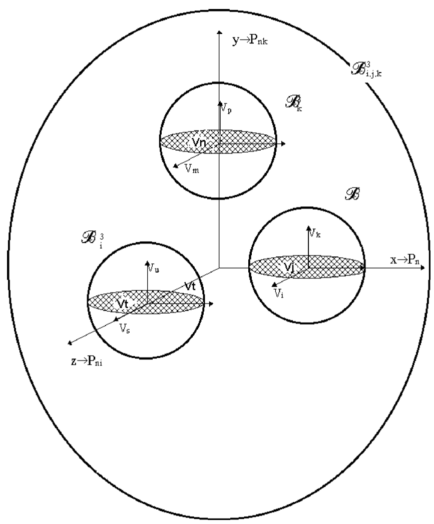

Fig.5. The model of 3-dimensional closed spaces $B^3_t \subset B^3_{i,j,k}$ $(t = i, j, k)$ for three orthogonal complete groups P_{ni}, P_{nj}, P_{nk}, where very $f \in P_{nt}$ is unit vector of B^3_t, and every group P_{nt} is unit vector of the space $B^3_{i,j,k}$

7 Making the decision

By definition the distribution of visual mass $\{m_{ij}\}$ on a matrix $4 \times 4(W_{4\times 4} \equiv W)$ is a pattern of the image $O(M)$ obtained as a result of action of D — pyramid at a stage of the initial description [10]. Let the set of filters $\{W_i\}$, belonging to D — pyramid, is applied to the whole area G of the image M. By virtue of (18) each distribution of masses $\{m_{ij}\}$ on a matrix W there is the standard for a subset of the images, presented by the value mij into it's own subarea $G_{ij} \subset G$. Let set $A = \{A_i\} : A_i = \{m_{ij}\}, A_i \longleftrightarrow W_i\}$ is th set of the image representations M on set $\{W_i\} \in D$. Then the following is correct

$$M_i \sim M_j \Leftrightarrow \{A_i\}_i = \{A_i\}_j. \tag{22}$$

Definition 8 *A subset of the images $\{M_k\} \subset M$ satisfying (22) is called as a class $Y_k(O_T)$ of images with the standard $O_T(M_k) = \{m_{ij}\}$ on a matrix W.*

For each distribution m_ij there corresponds a vector as the result of action of a M-pyramid to the initial description [10]. As two images $_i, M_j$ satisfying (22), are ϵ-close on the standard (are indiscernible) by use of the P-pyramid to the whole field of view first for them it is correct

$$M_i \sim M_j \Leftrightarrow O(M_i) = O(M_j). \tag{23}$$

Secondly, necessity of closeness of specification M_i, M_j with each other is natural.

Definition 9 *Let G_i — subarea of a sight field G. Then the application of a Q-pyramid to the image, determined in G_i with reception of a vector $M(G_i)$ is called as ϵ-exact decomposition of the image $M(G_i) \subset M(G)$ of level i.*

Hence the application of the Q-pyramid to the whole field G is decomposition of a level 1 (or 0). As corrections of the image M can be (in a general case) continued to a limit of the exactness (to pixel of the initial image) then:

1) we obtain multilevel hierarchy of piramidal decomposition where a universal element is the Q-pyramid;

2) the decomposition of the image begins as a whole (at first the Q-pyramid applies to the whole field G then to its subareas);

3) the decision making according to (22), (23) begins at immediately.

Statement 4 *Let F is (1). Then the set F is necessary and sufficient for authentic and ϵ-exact uncovering of a priori uncertainty in any subarea $G_i \subseteq G$ of a range of the image definition and minimal necessary for uncertainty uncovering with any given accuracy.*

The result is multilevel hierarchy

$$U(Q) = U_0(U_1(U_2(...U_k(Q))...), \tag{24}$$

where $U_k(Q) = \{M(G_k)\}$ is set of the image decomposition M determined in area $G, G_k \subseteq G; M(G_k) = \{\mu_i)_k$ is decomposition of the image M determined in a subarea G_k on set F of the Q-pyramid; $G_{k=0} \equiv G$.

Definition 10 *The hierarchy (24) is called as U-pyramid.*

Representation of the image for (22) is the set of structural elements $\{m_{ij}\}$ obtained as the result of action of P-pyramid [10]. Representation of the image for (23) is set $\{\mu_i\}$ obtained as the result of action of the M-pyramid. The description of the patter $O(M)$ is result of interaction of the operators $\{V_i\}$ ordered on a lattice $V(x, y)$. Besides the set $\{m_{ij}\}$ is set of absolute meanings of masses and set $\{\mu_i\}$ is set of relative one [10]. Therefore is correct

Consequence 6 *Let $\{\mu_i\}_1, \{\mu_i\}_2$ — representation of the images M_1, M_2 for which is fulfilled $\mu_0(M_1) = a,\ \mu_0(M_2)$ where a is constant. Then*

$$\begin{gathered} M_i \sim M_2 \longleftrightarrow \{\mu_1 : i \neq 0\}_i \sim \\ \{\mu_i : i \neq 0\}_2 \Leftrightarrow O(M_1) \Leftrightarrow O(M_2). \end{gathered} \tag{25}$$

Thus, if the conditions (22), (23) (first for representation $\{m_{ij}\}$, second for $\{\mu_i\}$) are necessary and sufficient for the image reference to a class then (25) is the necessary and sufficient condition of equivalence of the images, wich is correct no only for a pair of the images of one level of decomposition but also for different levels of the U-pyramid.

Property 2. Let M_i, M_j are images of levels i, j of the U-pyramid. Let $Y_k(O_T) \equiv Y_k \in Y$ is the images class on set of classes Y and for M_i, M_j it is correct (25). Then

$$M_i \sim M_j \Leftrightarrow (M_i, M_j) \in Y_k. \tag{26}$$

The conditions (25), (26) are necessary and sufficient conditions of invariancy of a pair of the images as on mass (invariancy under transformation of gomothety) and on a range of definition (invariancy under affine transformations of a kind similar deformations, for example, stratching or contraction).

Let $O_i(M), O_j(M)$ are patterns of the images M_i, M_j, not satisfying to a condition (25). By virtue of consequence 4 and 5 on set of patterns $\{O_i(M)\}$, presented on a matrix W, there are the subsets, satisfying to conditions (19), (20) and allotted with the standards according to condition (18). Besides each image M can be decomposed in accordance with lemma 5 into complete basic groups of family S_{nb} and as this decomposition is minimum the set of the descriptions (patterns) $O(P_{ni})$ will form a subset of classes according to criterion (18). And, if conditions (23), (25) are conditions of reference of the image M to a point class $Y_i(O_T)$ presented a unique ϵ-exact pattern $O(M)$ then condition

$$O_i(M) \sim O_j(M) \Leftrightarrow [(O_i(M), O_j(M)) \in ((O(P_v)) \bigvee O(\bar{P}_v)) \bigwedge P_c] \tag{27}$$

is the condition reference of the image to a fuzzy class $Y_r(O_T)$.

General property for pointed classes is their belonging to a space with basis (14).

Definition 11 *Space $B^n(V)$ with basis $\{V_i : i = \overline{1,15}\}$, determined on (14) is called as space of classes.*

Let us select some properties of space $B^n(V)$. Each point of space is a vector in basis (14) and as for any image conditions (22) are fulfilled the space is closed and has capacity $k(V)_{max} = P^{4\times 4}$ (where P is number of brightness gradation [12]). And the part of set $4 \times 4 {\times} P$ is a subset (family) of binary images $S(V, P_n)$

Consequence 7 *Let $B^n(V)$ is space with basis (14). Then $B^n(V)$ is closed*

$$S(P_s, P_n) \subseteq B^n(V) \subseteq 4 \times 4 \times P. \tag{28}$$

In correspondence with (28) mass of atom not more $PS(G_i)$ where $S(G_i)$ is area of a subregion $G_i \subseteq G$. In this case there are the lower and uppper bounds of pattern mass on a matrix 4×4

$$\begin{cases} \inf_{O_1} \mu_0(O_i) = e_a & (a) \\ \sup_{O_1} \mu_0(O_i) = 16 e_a, & (b) \end{cases} \tag{29}$$

where unit is the mass of atom ea = 1 of binary image. Then the space $B^n(V)$ is a normalized vector space for which Minkowski inequality is fulfilled and as $\mu_0(\sum V_i) \neq \sum \mu_0(V_i)$ and it is correct (29) this space is point Riemann space for which the next conditions exist [14]:

1) scalar product of two vectors a, b with components $\{a^i\}, \{b^i\}$ or $\{a_i\}, \{b_i\}$

$$ab = g_{ik} a^j d^k = g^{ik} a_i b_k \tag{30}$$

2) Length of a vector a is

$$\mid a \mid = +\sqrt{a^2}, \qquad a^2 = aa = g_{ik} a^i a^k = a^i a_k = g^{ik} a_i a_k, \tag{31}$$

where g_{ik} and g^{ik} belong to fundamental tensor of Riemann space and define its internal geometry.

Let the operators V_i and V_j correspond to vectors a and b accordingly. By definition the product $(V_i V_j)$ belongs to complete group and has mass $\mu_0(P_n)$ equal to 4 units. Therefore by virtue of (30) metric tensor is constant for any pairs of the operators. Then each complete group belongs to tangential space with Euclidean metric and sets local orthogonal basis of 3-dimensional Riemann space.

Lemma 6 *The space of classes $B^n(V)$ is Riemann point space with orthogonal local basis (14) where set $B_i^3(V)$ corresponds to set $\{P_{ni}\}$. As between sets $\{V_i\}$ on (14) and $\{F_i\}$ on (1) there is one-to-one correspondence it is correct*

Consequence 8 *Space $B^n(V)$ isomorphism to C^∞ — variety on set $\{F_i\}$ (i.e. $\{Q_{xy}^{i,j}\}$). By definition the initial representation of the image belongs to C^∞ — variety [12]. Then its section by filters from set F is a stratification of space on the set of tangential spaces. We shall name this set, following [15], as map of the image M of a level 0 (zero). Then the set of maps on levels collection (24) is the image atlas. Since variety is indefinitely smoothly any pair of maps of the next levels belonging the same subregion of a sight field satisfies to the requirement of C^∞ — coordination.*

Consequence 9 *The U-pyramid (24) with base representation on a Q-pyramid is C^∞ coordinated.*

As each filter from set F is the differential operator [12] it is correct.

Consequence 10 *The set F defines set of the unique geodesic lines of Riemann space $B^n(V)$.*

8 Conclusion

Final set of the operators (matrices) $\{V_i\} = V$ on the set of filters $\{Q_{xy}^{i,j}\}$, allowing to introduce concept of the image algebra has developed. It is proved, that the algebra of the image is the Boolean algebra, for which correspondence $< V;+,\cdot > \;\Leftrightarrow\; < V;Y;I >$ is fulfilled. Thus an opportunity of expansion of one — place operators final set, implemented by filters $Q_{xy}^{i,j}$, and opportunity to construct the predicates $P(\{V_j\})$, revealing any j-arity relations on the binary relations final set, have been introduced. As to each filter $\{Q_{xy}^{i,j}\}$, so to the operator $V_k \in V$, there corresponds differential structure and the result of action of each filter is an implementation of a measure, the algebra $A(V)$ is homological algebra.

Correspondence $\{\mu_i\} \longleftrightarrow \{V_i\}$ was proved. Thus the mathematical apparatus of synthesis of ϵ-exact images on set $\{V_i\}$ and operations (2), (4) was determined. The following concepts where introduced and forvalized: the image pattern, as of the ϵ-exact description of the image on set $\{V_i\} : M \cong O(M) \equiv P(\{V_i\})$; grammar of the image description (13); the descriptive image represententions.

Existence of groups and families of groups on set $\{V_i\}$ was proved and was developed methodology of their description and detection on the image, presented by set $\{\mu_i\}$. The next concepts were introduced and formalized: the graph on a lattice $V(x, y)$; predicates of coincidence and distinction. On the basis of last on set of groups concept of groups algebra $A(P) =< P; P_c, P_p >$ on a lattice $V(x, y)$ was given.

Methodology of constructing the authentic and ϵ-exact descriptive description of the image, presented by set $\{\mu_i\}$ was developed. Concept of a pattern skeleton of the image is formalized.

Properties (axioms) for set of complete groups on a lattice $V(x, y)$ are formalized. Concept of basic complete groups is introduced and existence of 3-dimensional basis on a subset of basic complete groups was proved. Moreover, if each complete group on triple of the operators sets three-dimensional basis of vector space, each triple of orthogonal complete groups is 3-dimesional basis of a new space, what is an association of three 3-dimensional vector spaces.

Questions of compactness of the image description were investigated. As a result concept of the image standard is formalized (thus a hypothesis of compactness was formalized). Minimality of the description for any ϵ-exact pattern and standard on final set of the operators, belonging to basic complete groups family. The principal aspects of the theory of the decision making in conditions of a priori ucertainty correctly by virtue fundamentality of transformations (1) for application to objects of any physical nature and any dimension were developed. The researches are fulfilled in Nizhniy Novgorod State Technical University (NSU), chair “Computing mashinery”, chairman corresponding member of the Academy of Science of Russia V.V. Kondrat’ev and in Tecnical University of Moldova (TUM), chair “Automatica”, chairman doctor habilitat of science tecnical, professor A.A. Gremalski Key

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