Chapter 7
Josephson Effect in SFNS Josephson Junctions

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Abstract The critical current, I_C, of Josephson junctions both in ramp-type (S-FN-S) and in overlap (SNF-FN-FNS, SN-FN-NS, SNF-N-FNS) geometries has been calculated in the frame of linearized Usadel equations (S–superconductor, F–ferromagnetic, N–normal metal). For the ramp-type structures, in which S electrodes contact directly the end walls of FN bilayer, it is shown that I_C may exhibit damping oscillations as a function of both the distance L between superconductors and thicknesses $d_{F,N}$ of ferromagnetic and normal layers. The conditions have been determined under which the decay length and period of oscillation of $I_C(L)$ at fixed d_F are of the order of decay length of superconducting correlations in the N metal, ξ_N, that is much larger than in F film. In overlap configurations, in which S films are placed on the top of NF bilayer, the studied junctions have complex SNF or SN electrodes (N or NF bilayer are situated under a superconductor). We demonstrate that in these geometries the critical current can exceed that in ramp-type junctions. Based on these results, the choice of the most practically applicable geometry is discussed.

7.1 Introduction

The existence of the oscillatory dependence of the critical current on the distance between superconducting electrodes reliably confirmed in a number of experiments

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using a variety of ferromagnetic materials and the types of Josephson junctions [1–16]. Promising use of π transitions, for which the critical current has a negative value, has been discussed in [17–21] for the implementation of qubits and for superconducting electronics. However, all these structures have some significant drawbacks, limiting their application.

The first of them is the smallness of the characteristic scale penetration of superconductivity in a ferromagnet. Indeed, analysis existing experimental data [1–16] shows that the value of exchange energy, H in ferromagnetic materials scales in between 850 and 2,300 K. Such large values of H lead to effective decay length, $\xi_{F1} \approx 1.2$–4.6 nm, and period of oscillations, $\xi_{F2} \approx 0.3$–2 nm, of thickness dependence of an SFS junction critical current, I_C. These values turned out to be much smaller compared to the decay length, $\xi_N \approx 10$–100 nm, in similar SNS structures. This fact makes it difficult to fabricate SFS junctions with reproducible parameters. It also leads to suppression of $I_C R_N$ product, thus limiting the cutoff frequency of the junctions. Since a search of exotic ferromagnetic materials with smaller value of H is challenging problem [16], one has to seek for another solutions.

Possible way to increase the decay length in a ferromagnetic barrier is the use of long-range proximity effect due to induced spin-triplet superconductivity [26–53] in structures with nonuniform magnetization. If magnetization of a ferromagnetic barrier is homogeneous, then only singlet component and triplet component with projection $S_z = 0$ of the total Cooper-pair spin are induced in the F region. These superconducting correlations are short-ranged, that is they extend into the F layer over a short distance of the order of $\xi_{F1} = \sqrt{D_F/H}$ in the diffusive case. However, in the case of inhomogeneous magnetization, for example in the presence of magnetic domain walls or in SF multilayer with noncollinear directions of magnetization of different F layers, a long-range triplet component (LRTC) with $S_z = \pm 1$ may appear. It decays into F region over distance $\xi_F = \sqrt{D_F/2\pi T_C}$ (here T_C is the critical temperature of S layer), which is by the factor $\sqrt{H/2\pi T_c}$ larger than ξ_{F1}. The latter property might lead to the long-range effects observed in some experiments [27, 28].

The transformation of decay length from ξ_{F1} to ξ_F might also take place in a vicinity of a domain wall even without generation of an odd triplet component [29–37]. This enhancement depends on an effective exchange field, which is determined by thicknesses and exchange fields of the neighboring domains. If a sharp domain wall is parallel [33, 36] or perpendicular to SF interface [37] and the thickness of ferromagnetic layers, $d_f \lesssim \xi_{F1}$, then for antiparallel direction of magnetization the exchange field effectively averages out, and the decay length of superconducting correlations becomes close to that of a single nonmagnetic N metal $\xi_F = \sqrt{D_F/2\pi T_C}$. It should be mentioned that for typical ferromagnetic materials ξ_F is still small compared to decay length $\xi_N \gtrsim 100$ nm of high conductivity metals such as Au, Cu, or Ag. This difference can be understood if one takes into account at least two factors. The first of them is that typical values of Fermi velocities in ferromagnetic materials (see, e.g., the analysis of experimental data done in [13, 14])