INVESTIGATION OF THE EFFECT OF AN EQUALIZING RESISTOR ON THE PARALLELING VOLTAGE SOURCES BY PROJECTIVE GEOMETRY

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Abstract

The method of current sharing of limited capacity voltage sources operating in parallel by equalizing resistors is considered. The effect of equalizing resistors is investigated on uniform distribution of relative values of currents when the actual loading corresponds to the capacity of a specific source. Necessary concepts for quantitative representation of operating regimes of sources are introduced using projective geometry.

1. Introduction

The basic problem of paralleled voltage sources is current sharing among these modules. Various approaches of current distribution are known [1]. In the most simple droop method, the equalizing resistors are used [2–4] including lossless passive elements [5]. Usually, the equality of module parameters, i.e., an open circuit voltage and internal resistance, is provided. Therefore, the distribution of currents means the equality of these currents. On the other hand, a scatter of module parameters, the use of voltage sources with different capacity determines the nonuniformity of current distribution. Therefore, it is natural to understand a uniform loading of sources in relative sense when the actual loading corresponds to the capacity of the source [6]. The analysis of this power supply system with a variable equalizing resistor for a given load leads to the introduction of some concepts for the quantitative representation of operating regimes and system parameters.

2. Analysis of paralleling voltage sources

Consider voltage sources E_1, E_2 in Fig. 1. Resistors R_{i1}, R_{i2} are internal resistances of these voltage sources; equalizing resistors R_{e1}, R_{e2} provide current distribution for a given load R_0.

The circuit in Fig. 1 is described by the following system of the equations

$$\begin{cases} E_1 = I_1(R_{i1} + I_1 R_{e1} + R_0) + I_2 R_0 \\ E_2 = I_1 R_0 + I_2(R_{i2} + I_2 R_{e2} + R_0) \end{cases}. \tag{1}$$

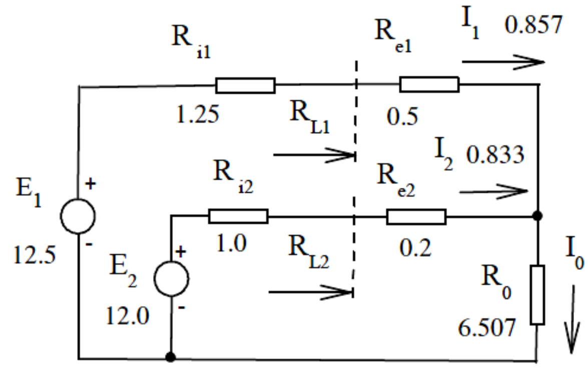

Fig. 1. Paralleling of voltage sources.

The variants of normalized parameters of a loading regime of the first source (used in different areas of electrical engineering, radio engineering, and power electronics) look like

$$m_1 = \frac{R_{L1}}{R_{i1}} = \frac{I_1^M}{I_1} - 1, \; J_1 = \frac{I_1}{I_1^M}, \tag{2}$$

here the maximum current of the source corresponds to the short circuit current

$$I_1^M = \frac{E_1}{R_{i1}}. \tag{3}$$

Similar considerations can be done for the second source:

$$m_2 = \frac{R_{L2}}{R_{i2}} = \frac{I_2^M}{I_2} - 1, \; J_2 = \frac{I_2}{I_2^M}.$$

Then we have

$$I_1 = \frac{I_1^M}{m_1 + 1} = \frac{E_1}{(m_1 + 1)R_{i1}}, I_2 = \frac{E_2}{(m_2 + 1)R_{i2}}.$$

Let us write expressions which associate the parameters of source loadings in the form $m_2(m_1)$ and $J_2(J_1)$ with the variable equalizing resistor R_{e1} for a given load. It follows from (1) that

$$E_2 = \frac{E_1}{(m_1 + 1)R_{i1}} R_0 + \frac{E_2}{(m_2 + 1)R_{i2}} (R_{i2} + R_{e2} + R_0).$$

Then we obtain

$$m_2(m_1) = \frac{\frac{R_{e2} + R_0}{R_{i2}} m_1 + \frac{R_{e2} + R_0}{R_{i2}} + \frac{E_1}{E_2}\frac{R_0}{R_{i1}}}{m_1 - \left(\frac{E_1}{E_2}\frac{R_0}{R_{i1}} - 1\right)} = \frac{\alpha m_1 + (\delta + \alpha + 1)}{m_1 - \delta}, \tag{4}$$

$$J_2(J_1) = -\frac{\delta + 1}{\alpha + 1} J_1 + \frac{1}{\alpha + 1}. \tag{5}$$

The plots of these dependences are presented in Fig. 2.

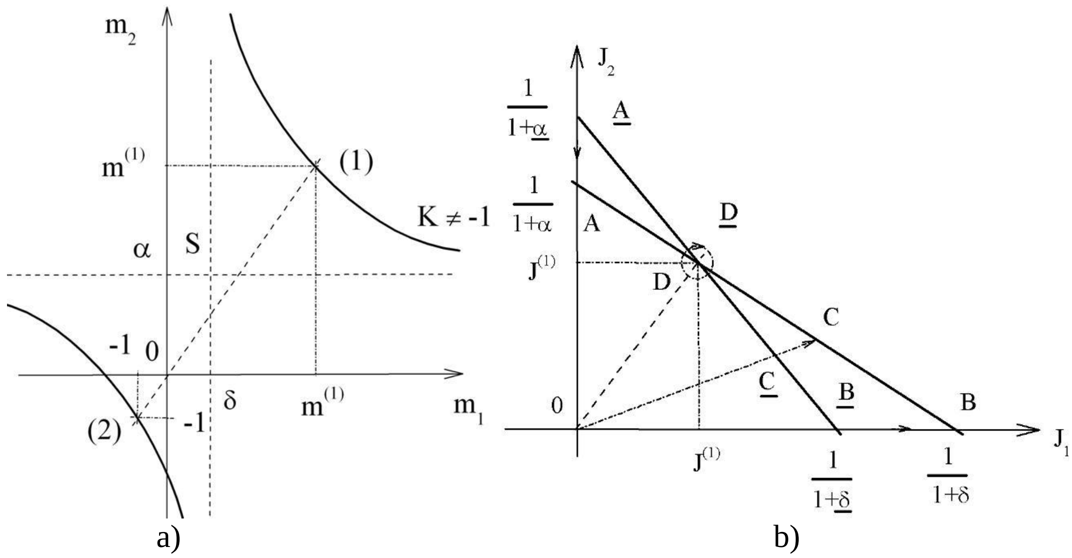

Fig. 2. Correlated changes of the source loadings parameters:
(a) as a hyperbole $m_2(m_1)$ for a given non-uniformity loading factor K;
(b) as a straight line $J_2(J_1)$ for two different systems.

Expression (4) corresponds to a hyperbole and (5) corresponds to a straight line. The desirable operating regime corresponds to the straight lines on these plots, $m_2 = m_1$, $J_2 = J_1$. The crossing of this straight line with the hyperbole gives two points of equal loading of sources $m^{(1)}, m^{(2)}$. The working area (load consumes energy) corresponds to the first point $m^{(1)}$. The second point corresponds to a condition when the voltage sources relatively equally consume energy. The points of equal loading are fixed points of the projective transformations $m_1 \to m_2, J_1 \to J_2$ as shown in Fig. 3. These transformations differ from the ones for the variable load case [6].

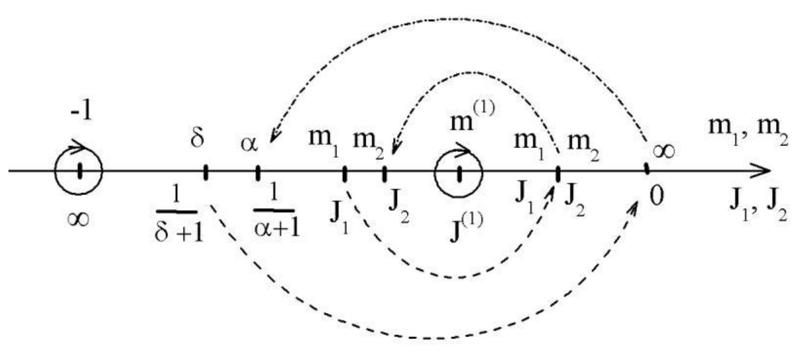

Fig. 3. Display of projective transformations of points $m_1 \to m_2,\ J_1 \to J_2$.

Let us find the fixed points $m = m_1 = m_2$. In this case, expression (4) leads to a quadratic one:

$$m^2 - (\delta + \alpha)m - (\delta + \alpha + 1) = 0 .$$

The solution gives two roots

$$m^{(1)} = \delta + \alpha + 1, \quad m^{(2)} = -1 \ , \tag{6}$$

that correspond to two fixed points of transformation (5):

$$J^{(1)} = \frac{1}{\delta + \alpha + 2}, \quad J^{(2)} = \infty . \tag{7}$$

For the second fixed point, the currents $I_2, I_1 \to \infty$. Though this case is not physically feasible, its mathematical description allows introducing some necessary characteristics of the circuit.

Similarly to [6], ***it is possible to introduce two concepts; one of them defines the circuit: how much the source loadings can differ. The second concept defines deviations of actual loadings from a fixed point in the relative form.*** In this case, it is possible to compare running regimes of different circuits.

For introduction of these characteristics, we use a number of concepts of projective geometry [7] applied in the circuit theory [8]. The characteristic of positioning of two points, concerning the chosen points (as the special case, it is the fixed points $m^{(1)}, m^{(2)}$) is the cross ratio of these four points

$$(m^{(2)} \ \ m_2 \ \ m_1 \ \ m^{(1)}) = \frac{m_2 - m^{(2)}}{m_2 - m^{(1)}} \div \frac{m_1 - m^{(2)}}{m_1 - m^{(1)}} , \tag{8}$$

where points $m^{(2)}, m^{(1)}$ are extreme or base. It is also known that the cross ratio corresponding to fixed points does not depend on values of running points m_1, m_2. Therefore, we accept for simplification of calculations that $m_1 = \infty$. Then

$$(m^{(2)} \ \ m_2(\infty) \ \ \infty \ \ m^{(1)}) = \frac{m_2(\infty) - m^{(2)}}{m_2(\infty) - m^{(1)}} = -\frac{\alpha + 1}{\delta + 1} = K \ ,$$

$$K = -\frac{\alpha + 1}{\delta + 1} = -\frac{E_2}{E_1} \cdot \frac{R_{i1}}{R_{i2}} \left(1 + \frac{R_{i2} + R_{e2}}{R_0} \right) < 0 . \tag{9}$$

The obtained expression is defined only by the circuit parameters and characterizes the ability of a circuit to equal loading of sources, which corresponds to the first introduced concept. This expression is referred to as the ***non-uniformity loading factor*** K. The negative value of K shows that points m_1, m_2 are located on different sides from fixed point $m^{(1)}$. For the variable load case, these points are located on the one side from the fixed point, and a similar factor takes place: $K > 0$ [6].

Equation (8), taking into account (9), gives the possibility to express the dependence $m_2(m_1)$ using only two parameters of the circuit, namely $m^{(1)}$ and K. Let us present (8) as

$$K = \frac{m_2 + 1}{m_2 - m^{(1)}} \div \frac{m_1 + 1}{m_1 - m^{(1)}} . \tag{10}$$

From here

$$m_2 = \frac{-\dfrac{1+Km^{(1)}}{1-K}m_1 + m^{(1)}}{m_1 - \dfrac{K+m^{(1)}}{1-K}} = \frac{-(1+Km^{(1)})m_1 + (1-K)m^{(1)}}{(1-K)m_1 - (K+m^{(1)})}, \quad \delta = \frac{K+m^{(1)}}{1-K}. \qquad (11)$$

3. Comparison of loading regime of different circuits

Let us obtain a normalized representation of dependence $m_2(m_1)$. For this purpose, we consider the cross ratios for variables m_1 and m_2, using their conformity, according to transformation (4). Therefore, the cross ratios are equal among themselves:

$$(-1\, m_1 \;\; m^{(1)}\; \delta) = (-1\, m_2 \;\; m^{(1)}\; \infty).$$

The cross ratio is a relative expression and gives a necessary normalization of the variables. Therefore, any variations of relative expressions for variables m_1 and m_2 are excluded.
Let us present each cross ratio as

$$(-1\, m_1 \;\; m^{(1)}\; \delta) = \frac{m_1+1}{m_1 - \dfrac{K+m^{(1)}}{1-K}} \div \frac{m^{(1)}+1}{m^{(1)} - \dfrac{K+m^{(1)}}{1-K}} = \frac{m_1+1}{m_1 - \dfrac{K+m^{(1)}}{1-K}} \cdot \frac{-K}{1-K},$$

$$(-1\, m_2 \;\; m^{(1)}\; \infty) = \frac{m_2+1}{m_2-\infty} \div \frac{m^{(1)}+1}{m^{(1)}-\infty} = \frac{m_2+1}{m^{(1)}+1}.$$

Then, we have the equation

$$\frac{m_2+1}{m^{(1)}+1} = \frac{m_1+1}{m_1 - \dfrac{K+m^{(1)}}{1-K}} \cdot \frac{-K}{1-K}.$$

The left side of this expression represents a normalized value and prompts how it is possible to write the similar value in the right side. Therefore,

$$\frac{m_2+1}{m^{(1)}+1} = \frac{-\dfrac{K}{1-K}\dfrac{m_1+1}{m^{(1)}+1}}{\dfrac{m_1+1}{m^{(1)}+1} - \dfrac{1}{1-K}}. \qquad (12)$$

Similarly, we have for (5)

$$\frac{J_2}{J^{(1)}} = \frac{J_1}{J^{(1)}K} - \frac{1-K}{K}. \qquad (13)$$

It should be noted that expressions (12) and (13) obviously set a deviation of running parameters of loading from the equal loading regime in the form of normalized values. But it is not enough for comparison of deviations for the circuits with different values of parameter K. Using the example of the most simple relation (13), we will show why it is so. Let us consider two circuits with different values of parameters $K, \underline{K}$, but with identical value $J^{(1)} = 1$. Characteristics of the circuits in the form of straight lines are presented in Fig. 2b. Loading regimes can be considered identical if the conformity of characteristic regime points takes place (shown by arrows) upon a change in the load. This follows from similarity principles [9]. Then, the projective transformation takes place and it is set by the center at point 0 and by three pairs of characteristic regime points: A, B, D and $\underline{A}, \underline{B}, \underline{D}$. The points $D, \underline{D}$ coincide and correspond to the fixed point $J^{(1)}$. The point of a running regime C corresponds to the point $\underline{C}$. For this projective transformation, the invariant in the form of a cross ratio is carried out $(A\,C\,D\,B) = (\underline{A}\,\underline{C}\,\underline{D}\,\underline{B})$.

Here, the points A, B and $\underline{A}, \underline{B}$ are base, and the points $D, \underline{D}$ are unit ones.

Therefore, this cross ratio can be accepted as an equal deviation of running points C, $\underline{C}$ from the unit points. Further, we display points A, C, D, B on the axis of currents J_1. Then, we obtain the deviation for the first source:

$$\Delta_1 = (0 \;\; J_1 \;\; J^{(1)} \;\; (1-K)J^{(1)}) = \frac{J_1}{J_1 - (1-K)J^{(1)}} \cdot K \;, \tag{14}$$

and the other form is

$$\Delta_1 = \left(0 \; J_1 \; J^{(1)} \; \frac{1}{\delta+1}\right) = \frac{J_1}{J_1 - \dfrac{1}{\delta+1}} \cdot \frac{-\alpha-1}{\delta+1} \tag{15}$$

The deviation for the second source is expressed similarly:

$$\Delta_2 = \left(0 \; J_2 \; J^{(1)} \; \frac{1}{\delta+1}\right).$$

Thus, the deviations include the circuit parameters and are not simply the normalized values as $J_1 / J^{(1)}$, $J_2 / J^{(1)}$.

With the aim to show the presented reasons, we will consider a special case, $K = -1$. Then, expressions (13) and (14) became

$$\frac{J_2}{J^{(1)}} = 2 - \frac{J_1}{J^{(1)}}, \; \Delta_1 = \frac{\dfrac{J_1}{J^{(1)}}}{2 - \dfrac{J_1}{J^{(1)}}} = \frac{2 - \dfrac{J_2}{J^{(1)}}}{\dfrac{J_2}{J^{(1)}}} = \frac{1}{\Delta_2}.$$

Thus, the deviations look like usual proportions and normalized values.

Taking into account the conformity between various definitions of loading parameters, the deviation is expressed in the invariant form through the cross ratio for the variable m_1

$$\Delta_1 = \left(0 \; J_1 \; J^{(1)} \; \frac{1}{\delta+1}\right) = (\infty \; m_1 \; m^{(1)} \; \delta) = \frac{m^{(1)} - \delta}{m_1 - \delta} = \frac{1+\alpha}{m_1 - \delta} \;. \tag{16}$$

The values of the deviations at the characteristic points are presented in Fig. 3. In particular, the deviation $\Delta^{(2)}$ for the second fixed point, $m^{(2)} = -1$, is equal to parameter K.

Again we will consider a special case, $K=-1$. Then, from (11), we obtain

$$m_2=\frac{\frac{m^{(1)}-1}{2}m_1+m^{(1)}}{m_1-\frac{m^{(1)}-1}{2}}=\frac{\delta m_1+m^{(1)}}{m_1-\delta},\ \alpha=\delta .$$

The points m_1, m_2 are symmetries concerning $m^{(1)}$ and bear the special name of harmonic conjugate points in projective geometry. In particular, the point $m_1=\delta$ corresponds to the point $m_2=\infty$ and vice versa. In addition, for the hyperbole in Fig. 2a, the center S will be on the straight line of equal loading.

Let us calculate deviations using (16). The deviation for the second fixed point, $\Delta^{(2)}=-1$, from the point $m^{(2)}$ concerning the base points is harmonic conjugate one to a unit point $m^{(1)}$.
In turn, the deviation for the second source is

$$\Delta_2=\left(\infty \quad m_2 \ m^{(1)} \ \delta\right)=\frac{1}{\Delta_1}.$$

We introduce the hyperbolic metric which defines the distances of the running regime points to the fixed point, $r_1=Ln\Delta_1$, $r_2=Ln\Delta_2$, $r_1=-r_2$. From here, the physical sense of value $K=-1$ follows: deviations of sources are equal in quantity, but are opposite in sign (this can be of practical use for all the equalizing resistor values).

Example. Consider the given circuit in Fig. 1.
Maximum currents of a source by (3), $I_1^M=10$, $I_2^M=12$. Parameters of the loading regime by (2), $m_1=10.66$, $J_1=0.0857$, $m_2=13.4$, $J_2=0.0694$.
Expressions of the loading regime of sources by (4), (5)

$$m_2=\frac{6{,}707m_1+12{,}13}{m_1-4{,}422},\quad J_2=-0{,}703J_1+0{,}13.$$

First fixed points by (6), (7), $m^{(1)}=12{,}13$, $J^{(1)}=0{,}0761$. Non-uniformity loading factor by (9), $K=-1{,}421$. Deviations for the sources by (16), $\Delta_1=1{,}2345$, $\Delta_2=0{,}802=1/1{,}246$.
The distances of the running regime points to the fixed point
$r_1=Ln\Delta_1=0{,}2106$, $r_2=Ln\Delta_2=-0{,}2206$.
The deviation for the second source is greater than for the first one.

4. Conclusions

We have proposed the concept of a nonuniformity loading factor for voltage sources which quantitatively characterizes the ability of a circuit for equal loading of sources. This factor makes it possible to compare different circuits.

We have introduced the concept of deviation of the running regime from the regime of

equal loading in a relative form for voltage sources. It gives the possibility to compare the deviations of the regime of the sources being parts of single or different supply systems.
Geometrical interpretation provides a validation for the introduction and definition of the proposed concepts.

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