On generalized multiplication groups of the commutative Moufang loops

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The multiplication groups of quasigroups, i.e. the groups generated by all left and right translations, represent an efficient tool in the theory of quasigroups (loops). For example, using the multiplications groups, the nilpotency of loops, in particular of commutative Moufang loops, is studied in [1,3,4], some invariants under the isostrophy of Bol loops are found in [5].
Belousov considered in [2] the groups, generated by all left, right and middle translations of a quasigroup, called the generalized multiplication group. He remarked that these groups are invariant under parastrophy of quasigroups, and found a set of generators for the stabilizer of a fixed element in the generalized multiplication group. The generalized multiplication groups and the generalized inner mapping groups are invariant under the isostrophy of loops [6]. Also, it was shown in [6] that the center $\overline{Z}(Q)$ of the generalized multiplication group GM(Q) of a loop Q defines a normal subloop, which consists of those elements of the loop, which are invariant under all mappings from the generalized inner mapping group $\overline{J}(Q)$ of Q.
The generalized multiplication group and the generalized inner mapping group of a commutative Moufang loop Q are considered in the present work. In particular it is proved that:

1. $\overline{J}(Q) \leq Aut(Q)$;

2. $GM(Q/\overline{Z}(Q)) \cong GM(Q)/\overline{Z}(Q)$;

3. $\overline{J}(Q/\overline{Z}(Q)) \cong \overline{J}(Q)/(\overline{Z}^*(Q) \cap \overline{J}(Q))$ and $\overline{Z}^*(Q) \cap \overline{J}(Q) \subseteq Z(\overline{J})$, where $\overline{Z}^*(Q)) = \{\alpha \in GM(Q) \mid \overline{Z}(Q)\alpha(x) = \overline{Z}(Q)x, \forall x \in Q\}$.

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