On non-isomorphic quasigroups of small order

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A non-empty set G is said to be a *groupoid* relatively to a binary operation denoted by $\{\cdot\}$, if for every ordered pair (a,b) of elements of G there is a unique element $ab \in G$.
A groupoid $(G,\cdot)$ is called a *quasigroup* if for every $a,b \in G$ the equations $a \cdot x = b$ and $y \cdot a = b$ have unique solutions.
A quasigroup $(G,\cdot)$ is called a *Ward quasigroup* if it satisfies the law $(a \cdot c) \cdot (b \cdot c) = a \cdot b$ for all $a,b,c \in G$.
A quasigroup $(G,\cdot)$ is called a *Cote quasigroup* if it satisfies the law $a \cdot (ab \cdot c) = (c \cdot aa) \cdot b$ for all $a,b,c \in G$.
A groupoid $(G,\cdot)$ is called a *Manin quasigroup* if it satisfies the law $a \cdot (b \cdot ac) = (aa \cdot b) \cdot c$ for all $a,b,c \in G$.

We consider the following problem:
Problem 1. How many non-isomorphic Ward quasigroups, Cote quasigroups and Manin quasigroups of order 3, 4, 5, 6 do there exist?
We elaborated algorithms for generating non-isomorphic Ward quasigroups, Cote quasigroups and Manin quasigroups of small order.The results established here are related to the work in ([1,2,3,4,5]). Applying the algorithms elaborated, we prove the following results:

Theorem 1. *There are exactly:*
- 1 non-isomorphic Ward quasigroup of order 3;
- 2 non-isomorphic Ward quasigroups of order 4;
- 1 non-isomorphic Ward quasigroup of order 5;
- 2 non-isomorphic Ward quasigroups of order 6.

Theorem 2. *There are exactly:*
- 3 non-isomorphic Cote quasigroups of order 3;
- 4 non-isomorphic Cote quasigroups of order 4;
- 2 non-isomorphic Cote quasigroups of order 5;
- 3 non-isomorphic Cote quasigroups of order 6.

Theorem 3. *There are exactly:*
- 3 non-isomorphic Manin quasigroups of order 3;
- 4 non-isomorphic Manin quasigroups of order 4;
- 4 non-isomorphic Manin quasigroups of order 5;
- 3 non-isomorphic Manin quasigroups of order 6.

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