On the generalized factorization of functions in weighted spaces

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In this paper the notion of factorization of functions with respect to contour Γ in the spaces $L_p(\Gamma,\rho)$ is presented [1]. The main result of the paper is the determination of some classes of functions that allow a factorization, as well as the application of factorization in studying of singular integral operators with measurable and bounded coefficients.
Let Γ be a closed Lyapunov contour which bounds the domain G^+. By G^- we denote the domain which complements $G\bigcup\Gamma$ to the whole plane. Assume that $0 \in G^+$ and $\infty \in G^-$.
Let $L_p^+(\Gamma,\rho) = P(GL_p(\Gamma,\rho))$, $L_p^-(\Gamma,\rho) = Q(GL_p(\Gamma,\rho)) + c$, $c \in \mathbb{C}$. **Definition.** The generalized factorization of function $a \in GL_\infty(\Gamma)$ with respect to contour Γ in the space $L_p(\Gamma,\rho)$ is its representation in the form

$$a(t) = a_-(t)t^\kappa a_+(t),$$

where $\kappa \in \mathbb{Z}$ and the factors $a_\pm$ satisfy the follwing conditions:
1) $a_- \in L_p^-(\Gamma,\rho)$, $a_+ \in L_q^+(\Gamma,\rho^{1-q})$, $a_-^{-1} \in L_q^-(\Gamma,\rho^{1-q})$,
$a_+^{-1} \in L_p^+(\Gamma,\rho)$ $(p^{-1} + q^{-1} = 1)$;
2) the operator $a_+^{-1}Pa_-^{-1}I$ is bounded in the space $L_p(\Gamma,\rho)$.
Denote by $R^+L_\infty(\Gamma)$ the set of measurable real functions on the contour Γ verifying the conditions $0 < es\sin_{t\in\Gamma} f\, a(t)$, $ess\sup_{t\in\Gamma} a(t) < \infty$.

Theorem 1. $R^+L_\infty(\Gamma) \subset Fact_{p,\rho}(\Gamma)$. *If* $a \in R^+L_\infty(\Gamma)$, *then*

$$a(t) = a_-(t) \cdot a_+(t),$$

where $a_+(t) = \exp(P\ln a)(t)$, $a_-(t) = \exp(Q\ln a)(t)$. *In addition, the functions* $a_\pm$ *verify the conditions:* $a_+^{\pm 1} \in L_\infty^+(\Gamma)$ *and* $a_-^{\pm 1} \in L_\infty^-(\Gamma)$.

Denote by $Nt_{p,\rho}(\Gamma)$ the set of measurable bounded functions $a(t)$ on the contour Γ for which the operator $A = aP + Q$ is Noetheran in $L_p(\Gamma,\rho)$.

Theorem 2. $Nt_{p,\rho}(\Gamma) = Fact_{p,\rho}(\Gamma)$.

Bibliography

[1] Gohberg, N. Krupnik, *Introduction to the theory of one-dimensional singular integral operators*, Vols. I.- Birkhäuse Verlag, Basel, 1992.