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Canonical forms for cubic differential systems with affine real invariant straight lines of total parallel multiplicity six and configurations $\big(2(m), 2(n), 1, 1\big)$

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We consider the real cubic differential system

$$\begin{cases} \dot{x} = P_0 + P_1(x,y) + P_2(x,y) + P_3(x,y) \equiv P(x,y), \\ \dot{y} = Q_0 + Q_1(x,y) + Q_2(x,y) + Q_3(x,y) \equiv Q(x,y), \ \gcd(P,Q) = 1, \end{cases} \tag{1}$$

and the vector field $\mathbb{X} = P(x,y)\frac{\partial}{\partial x} + Q(x,y)\frac{\partial}{\partial y}$ associated to system (1).
A straight line $l \equiv \alpha x + \beta y + \gamma = 0$, $\alpha, \beta, \gamma \in \mathbb{C}$ is invariant for (1) if there exists a polynomial $K_l \in \mathbb{C}[x,y]$, $\deg(K_l) \leq 2$ such that the identity

$$\alpha P(x,y) + \beta Q(x,y) \equiv (\alpha x + \beta y + \gamma) K_l(x,y), \ (x,y) \in \mathbb{R}^2 \tag{2}$$

holds. The polynomial $K_l(x,y)$ is called cofactor of the invariant straight line l. If m is the greatest natural number such that l^m divides $\mathbb{X}(l)$ then we say that l has parallel multiplicity m. By present a great number of works have been dedicated to the investigation of polynomial differential systems with invariant straight lines.
The classification of all cubic systems with the maximum number of invariant straight lines, including the line at infinity, and taking into account their geometric multiplicities, is given in [1], [4], [5].
The cubic systems with exactly eight and exactly seven distinct affine invariant straight lines have been studied in [4], [5]; with invariant straight lines of total geometric (parallel) multiplicity eight (seven) - in [2], [3], [8], and with six real invariant straight lines along two (three) directions - in [6], [7]. In [9] it was shown that in the class of cubic differential systems the maximal multiplicity of an affine real straight line is seven.
In this paper are obtained canonical forms for cubic differential systems with affine real invariant straight lines of total parallel multiplicity six and configurations $\big(2(m), 2(n), 1, 1\big)$. It was proved
Theorem. *Assume that a cubic system* (1) *possesses affine real invariant straight lines of total parallel multiplicity six and of four distinct directions. At least two of these lines are multiplicity one. Then via an affine transformation and time rescaling this system can be brought to one of the three systems:*

$$1) \begin{cases} \dot{x} = x(x+1)(a+bx+y), \\ \dot{y} = y(y+1)\big(a+(a+b)x+(1-a)y\big), \\ ab(a+b)(2a-b-1) \neq 0, (a-1)(b-a+1) < 0, a, b \in R, \end{cases}$$

$\big(l_1 = x, l_2 = x+1, l_3 = y, l_4 = y+1, l_5 = y-x, l_6 = y - \frac{b}{a-1}x\big);$

2) $\begin{cases} \dot{x} = x(x+1)\big(a + (2a-1)x + y\big), \\ \dot{y} = y(y+1)\big(a + x + (2a-1)y\big), \\ a(a-1)(3a-1)(3a-2) \neq 0, a \in R, \end{cases}$

$\big(l_1 = x, l_2 = x+1, l_3 = y, l_4 = y+1, l_5 = y-x, l_6 = y+x+1\big);$

3) $\begin{cases} \dot{x} = x^2(bx+y), \\ \dot{y} = y^2\big((a+b)x + (1-a)y\big), \\ ab(b+1)(a+b)(2a-b-1)(a-2b-2) \neq 0, \\ (a-1)(b-a+1) < 0, a, b \in R, \end{cases}$

$\big(l_1 = l_2 = x, l_3 = l_4 = y, l_5 = y-x, l_6 = y - \frac{b}{a-1}x\big).$

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