The sufficient center conditions for some classes of bidimensional cubic differential systems

Iurie Calin[1,2] Valeriu Baltag[1]

[1]*Institute of Mathematics and Computer Science,*
[2]*Moldova State University, Chişinău, Republic of Moldova*
e-mail: iucalin@yahoo.com, vbaltag99@gmail.com

,

Let us consider the cubic system of differential equations

$$\begin{aligned} \frac{dx}{dt} &= P_1(x,y) + P_2(x,y) + P_3(x,y) = P(x,y), \\ \frac{dy}{dt} &= Q_1(x,y) + Q_2(x,y) + Q_3(x,y) = Q(x,y), \end{aligned} \tag{1}$$

where $P_i(x,\ y),\ Q_i(x,\ y)$ are homogeneous polynomials of degree i in x and y with real coefficients. The following $GL(2,\mathbb{R})$-comitants [1] have the first degree with respect to the coefficients of the system (1):

$$R_i = P_i(x,y)y - Q_i(x,y)x,\ S_i = \frac{1}{i}\left(\frac{\partial P_i(x,y)}{\partial x} + \frac{\partial Q_i(x,y)}{\partial y}\right),\ i = 1,2,3. \tag{2}$$

The definition of the transvectant of two polynomials is well known in the classical invariant theory [2].

Definition. *Let $f(x,y)$ and $\varphi(x,y)$ be homogeneous polynomials in x and y with real coefficients of the degrees $\rho \in \mathbb{N}^*$ and $\theta \in \mathbb{N}^*$, respectively, and $k \in \mathbb{N}^*$. The polynomial*

$$(f,\varphi)^{(k)} = \frac{(\rho-k)!(\theta-k)!}{\rho!\theta!}\sum_{h=0}^{k}(-1)^h\binom{k}{h}\frac{\partial^k f}{\partial x^{k-h}\partial y^h}\frac{\partial^k \varphi}{\partial x^h \partial y^{k-h}}$$

is called the transvectant of the index k of the polynomials f and φ.

Remark. *If the polynomials f and φ are $GL(2,\mathbb{R})$-comitants of the degrees $\rho \in \mathbb{N}^*$ and $\theta \in \mathbb{N}^*$, respectively, for the system (1), then the transvectant of the index $k \le \min(\rho,\theta)$ is a $GL(2,\mathbb{R})$-comitant of the degree $\rho+\theta-2k$ for the system (1). If $k > \min(\rho,\theta)$, then $(f,\varphi)^{(k)} = 0$.*

By using the transvectants for the system (1) the following $GL(2,\mathbb{R})$-invariants were constructed:

$$I_1 = S_1, \qquad I_2 = (R_1,R_1)^{(2)}, \qquad I_4 = (R_1,S_3)^{(2)}.$$

The system (1) can be written in the following coefficient form:

$$\begin{aligned} \frac{dx}{dt} &= cx + dy + gx^2 + 2hxy + ky^2 + px^3 + 3qx^2y + 3rxy^2 + sy^3, \\ \frac{dy}{dt} &= ex + fy + lx^2 + 2mxy + ny^2 + tx^3 + 3ux^2y + 3vxy^2 + wy^3. \end{aligned} \tag{3}$$

In this paper only the cubic differential systems (1) (or (2)) with $S_2 \equiv 0$, $I_1 = 0$ and $I_2 > 0$ were considered.

By using a center-affine transformation and time scaling the system (3) with $I_1 = 0$, $I_2 > 0$ and $S_2 \equiv 0$ can be reduced to the form:

$$\begin{aligned} \frac{dx}{dt} &= y + gx^2 + 2hxy + ky^2 + px^3 + 3qx^2y + 3rxy^2 + sy^3, \\ \frac{dy}{dt} &= -x + lx^2 - 2gxy - hy^2 + tx^3 + 3ux^2y - 3qxy^2 + wy^3. \end{aligned} \tag{4}$$

In this paper the sufficient center conditions for the origin of coordinates of the phase plane for the cubic differential system with $I_1 = 0$, $I_2 > 0$, $S_2 \equiv 0$ were established.

Theorem. *The system* (4) *has singular point of the center type in the origin of the coordinates, if* $w = -p - r - u$ $(I_4 = 0)$ *and one of the series of the conditions*

$$\begin{aligned} &1) \quad p + u = 0; \\ &2) \quad k - l = g + h = r + u = s + t = 0; \\ &3) \quad k + l = g - h = r + u = s + t = 0 \end{aligned}$$

is fulfilled.

Acknowledgement. *This research is partially supported by the grant 15.817.02.03F from SCSTD of ASM*

Bibliography

[1] K. S. Sibirsky, Introduction to the Algebraic Theory of Invariants of Differential Equations. Manchester University Press, 1988.

[2] G. B. Gurevich, Foundations of the Theory of Algebraic Invariants. Noordhoff, Groningen, 1964.

Intervals of linear stability of geometrical parameters in the restricted eight bodies problem with incomplete symmetry

Elena Cebotaru

Technical University of Moldova, Chişinău, Republic of Moldova
e-mail: elena.cebotaru@mate.utm.md

We consider the Newtonian restricted eight bodies problem with incomplete symmetry. We investigate the linear stability of this configuration by some numerical methods. For geometric parameter intervals of stability and instability are found, the corresponding theorem are formulated and proved. All relevant and numerical calculation are done with the computer algebra system Mathematica.

Keywords: Newtonian problem; differential equation of motion; configuration; particular solutions; equilibrium points; linear stability.

Bibliography

[1] E. A. Grebenikov, *On a mathematical problems of homographic dynamics*, Moscow: MAKS Press, 2010, 253 p. (in Russian)

[2] E. A. Grebenikov, D. Kozak-Skovorodkina, M. Yakubyak, *Computer algebra methods in the many-body problem*, Second edition, revised and supplemented, Moscow: The Peoples' Friendship University of Russia, 2002, 211 p. ISBN: 5-209-01451-7. (in Russian)