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ROCKER MECHANISM FROM CLASSICAL TO MODERN KINEMATICAL ANALYSIS

Iulian Malcoci*, ORCID ID: 0000-0002-0263-5232,
Marin Guțu, ORCID ID: 0000-0002-3745-0289

Technical University of Moldova, 168, Ștefan cel Mare Bd., MD-2004, Chișinău, Republic of Moldova
*Corresponding author: Iulian Malcoci, *iulian.malcoci@bpm.utm.md*

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Abstract. Kinematical mechanism analysis is usually used for motion study or to simulate and analyse the movement of mechanical assemblies and the whole mechanism. In this paper we will consider kinematical analysis regarding the rocker mechanism which is commonly used in a cross planer machine tool without taking into account forces that cause the mechanism motion. The authors have used a calculation model and a calculation algorithm that allowed the definition of kinematic parameters of the mechanism, including linear velocities and acceleration, angular speeds and acceleration. The calculations were performed using graphic-analytical methods and computational method using SolidWorks Motion Analysis software which provides a special module for quick analysis with a visual representation of characteristic data mechanism for different members position as a diagram representation of certain parameters throughout the researched motion cycle, which are only some of the possible areas of analysis. The resulted calculations are presented as numerical values and chart plots for comparison and validation.

Keywords: *rocker mechanism, graphic-analytical method, motion simulation, velocity distribution, acceleration distribution, SolidWorks Motion Analysis.*

Introduction

Kinematical study in this paper will be performed regarding rocker mechanism (Figure 1) with following initial data, drawing scale factor $\mu_l = 0{,}001\left[\dfrac{m}{mm}\right]$: - crank angular speed $\omega_1 = 12{,}56[s^{-1}] = \text{constant}$; - the lengths $O_1A = 0{,}0225[m]$; $O_1O_2 = 0{,}06[m]$; $O_2B = 0{,}092[m], BC = 0{,}025[m]$;

$$a = 0{,}089[m].$$

To determine velocity and acceleration distribution we will use *the velocity plan and the vector equations method* at certain mechanism position when $\varphi = 45°$.

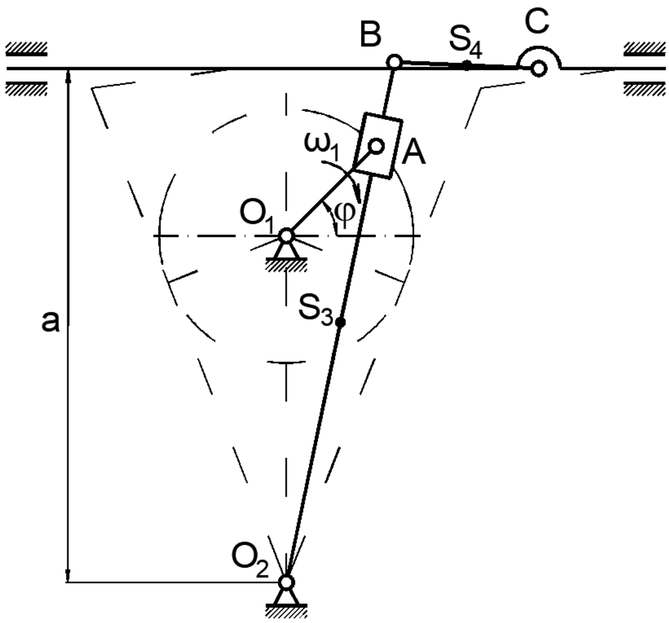


Figure 1. Studied rocker mechanism [3].

Velocity distribution (velocity plan and the vector equations method)

As you can see (Figure 1), point *A* performs compound motion: rotation motion around the axis O_1 with angular speed ω_1, translation motion along O_2B and balancing with O_2B around axis O_2 with angular speed ω_3. For this reason we have to determine 2 velocities for point *A* wich belong to the crank 1 and for the point A_3 wich belong to the coulisse 3. First of all, we can determine point *A* velocity using "Eq.(1)" [1, 2]:

$$(\perp O_1A,\ \to\omega_1):\ v_A = \omega_1 \cdot O_1A\,[m/s] \tag{1}$$

$$v_A = 12{,}56\ \ [s^{-1}]\cdot 0{,}0225\,[m] = 0{,}28\ [m/s].$$

To apply the velocity plan and the vector equations method and start construction of velocity plan at beginning we adopt velocity scale factor μ_V, so that velocity $\bar{v}_A$ does not exceed 50[mm] in velocity plane (Figure 2):

$$\mu_V = \frac{v_A}{50[mm]} = \frac{0{,}28[m/s]}{50[mm]} \cong 0{,}005\left[\frac{m/s}{mm}\right].$$

The next step, we pass the point *A* velocity through the chosen scale factor μ_V:

$$\overline{pa} = \frac{v_A}{\mu_V} = \frac{0{,}28[m/s]}{0{,}005\left[\dfrac{m/s}{mm}\right]} \cong 56\ \ [mm].$$

This method is a graphic-analytical method and it is based on Euler's velocities equations for plane-parallel motion "Eq.(2)" [2,4,5]:

$$\begin{matrix} A_3 \to A \\ A_3 \to O_2 \end{matrix}\ \begin{cases} (\perp O_2A)\ \underline{\bar{v}}_{A_3} = \underline{\underline{\bar{v}}}_A + \underline{\bar{v}}_{A_3A}\ (//O_2A) \\ \bar{v}_{A_3} = \bar{v}_{O_2} + \underline{\bar{v}}_{A_3O_2}\ (\perp O_2A) \end{cases} \tag{2}$$

where first equation represents motion of point A_3 against point *A* and second equation the motion of point A_3 against point O_2. Because point O_2 is a fixed result that $\left|\bar{v}_{O_2}\right| = 0$ and we can consider that $\bar{v}_{A_3} = \bar{v}_{A_3O_2}$. In this case we get a rotational motion of point A_3 around O_2 and for rotation motion we know $\bar{v}_{A_3O_2} \perp O_2A.$ On the other hand point A_3 (which belong to the pistons) perform a translation motion along rocker O_2A for this reason $\bar{v}_{A_3A} // O_2A$. The different velocities are represented in an arbitrary plan as vectors, with the modules reduced to the scale factor, *velocity scale factor* μ_V. In this plane, called *the velocity plan*, the null speed point is called *the velocity pole* and is marked with *p*.

In the velocity plans, the relationships like "Eq.(2)" are used, which are vector equations and it is solved graphically by constructing the velocity plan (Figure 2). Further using this method we will determine the velocity distribution for the rocker mechanism for a certain mechanism position $\varphi = 45°$ (Figure 1).

To determine point A_3 velocity we will use similarity report between mechanism scheme (Figure 1) and velocity plan (Figure 2):

$$\frac{O_2A}{O_2B} = \frac{pa_3}{pb} \Rightarrow pb = pa_3\frac{O_2B}{O_2A} \cong 47\ [mm]\cdot\frac{92\ [mm]}{77\ [mm]} \cong 56\ [mm].$$

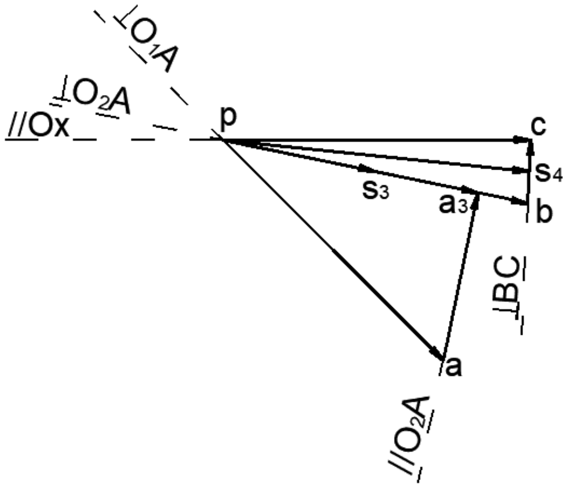


Figure 2. The velocity plan.

Applying Euler's velocities equations for point *C* plane-parallel motion against point *B* we obtain "Eq.(3)"

$$B \to C : (//Ox)\ \underline{\bar{v}_C} = \underline{\underline{\bar{v}_B}} + \underline{\bar{v}_{CB}}\ (\perp BC) \quad (3)$$

To determine point S_3 and S_4 velocities (weight centres) we will use theory of similarity and put point s_3 and s_4 at the middle distance of segment $\overline{pb}$ and $\overline{bc}$ in the velocities plan (Figure 2). After we finish constructing velocities plan (Figure 2) we can determine velocities distributions.

Velocities distribution we will determine from velocity plans:

$$v_{S_3} = ps_3 \cdot \mu_V = 28\ [mm] \cdot 0{,}005 \left[\frac{m/s}{mm}\right] = 0{,}140 [m/s];$$

$$v_{S_4} = ps_4 \cdot \mu_V = 55 [mm] \cdot 0{,}005 \left[\frac{m/s}{mm}\right] = 0{,}275 [m/s];$$

$$v_{A_3} = pa \cdot \mu_V = 47\ [mm] \cdot 0{,}005 \left[\frac{m/s}{mm}\right] = 0{,}235 [m/s];$$

$$v_{A_3A} = aa_3 \cdot \mu_V = 31\ [mm] \cdot 0{,}005 \left[\frac{m/s}{mm}\right] = 0{,}155 [m/s];$$

$$v_B = pb \cdot \mu_V = 56 [mm] \cdot 0{,}005 \left[\frac{m/s}{mm}\right] = 0{,}280 [m/s];$$

$$v_C = pc \cdot \mu_V = 55 [mm] \cdot 0{,}005 \left[\frac{m/s}{mm}\right] = 0{,}275 [m/s];$$

$$v_{CB} = bc \cdot \mu_V = 12 [mm] \cdot 0{,}005 \left[\frac{m/s}{mm}\right] = 0{,}060 [m/s];$$

$$\omega_3 = \frac{v_B}{O_2B} = \frac{0{,}28 [m/s]}{0{,}092 [m]} = 3{,}043 \left[s^{-1}\right];$$

$$\omega_4 = \frac{v_{CB}}{BC} = \frac{0{,}06 [m/s]}{0{,}025 [m]} = 2{,}4 \left[s^{-1}\right].$$

Accelerations distribution (acceleration plan and the vector equations method)

First acceleration which we can find is point *A* acceleration (rotation motion around axis O_1): $\bar{a}_A = \bar{a}_A^n + \bar{a}_A^\tau$, but $\omega_1 = const. \Rightarrow \varepsilon_1 = 0 \Rightarrow a_A^\tau = 0 \Rightarrow \bar{a}_A = \bar{a}_A^n \,(//O_1A, A \rightarrow O_1)$

$$\left|\bar{a}_A^n\right| = \omega_1^2 \cdot O_1A = 12{,}56^2\left[s^{-2}\right] \cdot 0{,}0225[m] = 3{,}54\left[m/s^2\right].$$

To apply the acceleration plan and the vector equations method and start construction of acceleration plan at beginning we adopt velocity scale factor μ_a, so that acceleration $\bar{a}_A$ does not exceed 70[mm] in velocity plane (Figure 3):

$$\mu_a = \frac{a_A}{70[mm]} = \frac{3{,}54\left[m/s^2\right]}{70[mm]} = 0{,}05\left[\frac{m/s^2}{mm}\right].$$

The next step, we pass the point *A* acceleration through the chosen scale factor μ_a:

$$\overline{\pi a} = \frac{a_A}{\mu_a} = \frac{3{,}54[m/s^2]}{0{,}05\left[\frac{m/s^2}{mm}\right]} \cong 71[mm].$$

This method is a graphic-analytical method and it is based on Euler's velocities equations for plane-parallel motion “Eq.(2)” [4 - 6]:

$$\begin{matrix} A_3 \rightarrow A \\ A_3 \rightarrow O_2 \end{matrix}\left\{\begin{aligned} &a_{A_3} = \underline{\underline{\bar{a}_A}} + \underline{\underline{\bar{a}^k_{A_3A}}} + \underline{\bar{a}^r_{A_3A}}\,(//O_2A) \\ &\bar{a}_{A_3} = \bar{a}_{O_2} + \underline{\underline{\bar{a}^n_{A_3O_2}}} + \underline{\bar{a}^\tau_{A_3O_2}}\,(\perp O_2A) \end{aligned}\right. \tag{4}$$

Coriolis and normal acceleration determination:

$$a^k_{A_3A} = 2 \cdot \omega_3 \cdot v_{A_3A} = 2 \cdot 3{,}043\left[s^{-1}\right] \cdot 0{,}155[m/s] = 0{,}943\left[m/s^2\right] /\ \mu_a \ \rightarrow \cong 19[mm],$$

$$a^n_{A_3O_2} = \frac{v^2_{A_3}}{l_{O_2A}} = \frac{v^2_{A_3}}{O_2A \cdot \mu_l} = \frac{0{,}235^2\,[m/s]^2}{77[mm] \cdot 0{,}001\left[\frac{m}{mm}\right]} = 0{,}717\left[m/s^2\right] / \mu_a \rightarrow \cong 14[mm].$$

To determine the direction of Coriolis acceleration, it is necessary to rotate the vector $\overline{aa_3}$ relative to the speed v_{A_3A} by 90° in the direction of the angular velocity ω_3 (Figure 3).

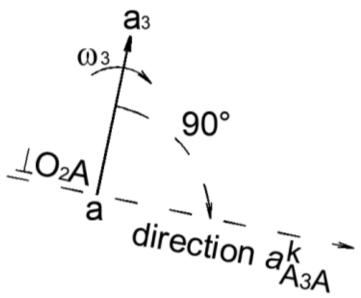


Figure 3. Coriolis acceleration direction.

The relative acceleration vector $a^{r}_{A_3A}$ of point A_3 in relation to the center of the hinge A is parallel to O_2A. The tangential acceleration vector $a^{\tau}_{A_3O_2}$ of point A_3 relative to O_2 is perpendicular to O_2A.

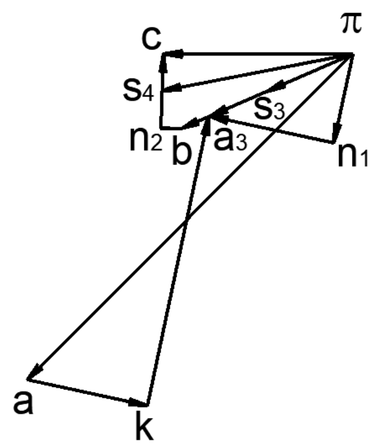


Figure 4. The acceleration plans.

We solve graphically equations 4 in order to build the acceleration plan (Figure 4), where π is the accelerations pole (the point where all accelerations are equal to zero).

To determine point A_3 acceleration we will use similarity report between mechanism scheme (Figure 1) and acceleration plan (Figure 4):

$$\frac{O_2A}{O_2B}=\frac{\pi a_3}{\pi b}\Rightarrow \pi b=\pi a_3\frac{O_2B}{O_2A}\cong 24[mm]\cdot\frac{92[mm]}{77[mm]}\cong 29[mm].$$

Next we can determine point C acceleration using a graphic-analytical method and Euler's velocities equations for plane-parallel motion "Eq.(5)", when we consider that point B moves against point A:

$$C\rightarrow B:(//Ox)\ \overline{a}_C=\underline{\underline{\overline{a}_B}}+\underline{\underline{\overline{a}^{n}_{CB}}}+\underline{\overline{a}^{\tau}_{CB}} \tag{5}$$

where normal acceleration $a^{n}_{CB}=\frac{v^2_{CB}}{BC}=\frac{0{,}06^2\,[m/s]^2}{0{,}025[m]}=0{,}144\left[m/s^2\right]/\mu_a\cong 3[mm]$.

After we have finished constructing acceleration plan (Figure 4) we can determine acceleration distributions:

$$a_{S_3}=\pi s_3\cdot\mu_a=14[mm]\cdot 0{,}05\left[\frac{m/s^2}{mm}\right]=0{,}7\left[m/s^2\right];$$

$$a_{S_4}=\pi s_4\cdot\mu_a=30[mm]\cdot 0{,}05\left[\frac{m/s^2}{mm}\right]=1{,}5\left[m/s^2\right];$$

$$a^{\tau}_{A_3A}=n_1a_3\cdot\mu_a=20[mm]\cdot 0{,}05\left[\frac{m/s^2}{mm}\right]=1{,}0\left[m/s^2\right];$$

$$a^{\tau}_{CB}=n_2c\cdot\mu_a=12[mm]\cdot 0{,}05\left[\frac{m/s^2}{mm}\right]=0{,}6\left[m/s^2\right];$$

$$a_C=\pi c\cdot\mu_a=29[mm]\cdot 0{,}05\left[\frac{m/s^2}{mm}\right]=1{,}45\left[m/s^2\right];$$

$$\varepsilon_3 = \frac{a^{\tau}_{A_3A}}{O_2A\cdot\mu_l} = \frac{1\left[m/s^2\right]}{77\left[mm\right]\cdot 0{,}001\left[\dfrac{m}{mm}\right]} = 12{,}98\left[s^{-2}\right]; \quad \varepsilon_4 = \frac{a^{\tau}_{CB}}{BC} = \frac{0{,}6\left[m/s^2\right]}{0{,}025\left[m\right]} = 24\left[s^{-2}\right]$$

Kinematical analysis using SolidWorks Motion Analysis

With the development of computer-aided engineering software, applications for kinematic analysis of mechanisms have been developed. These are usually integrated into 3D modeling software. At the same time, there are software programs that greatly facilitate analytical calculations such as Mathcad, Mathlab, etc. A kinematic analysis of the crank mechanism using the Mathcad applet is presented in the paper [7]. In this chapter it is presented the kinematic analysis of the rocker mechanism using SolidWorks Motion Analysis (available with the SolidWorks Motion add-in) [8, 9]. The analysis options of this software allow motion simulation of the complex assemblies such as the robotic arm [11]. It enables engineers to simulate design performance and identify and address potential design problems before prototyping and production [12]. Motion simulation provides complete, quantitative information about the kinematics – including position, velocity, and acceleration, and the dynamics — including joint reactions, inertial forces, and power requirements, of all the components of a moving mechanism. Often of a great additional importance, the results of motion simulation can be obtained virtually at no additional time expense, because everything needed to perform motion simulation has been defined in the CAD assembly model already [10]. Engineers can represent simple mechanisms such as 2D mechanisms. Although these are difficult and time-consuming to analyze by hand, they do possess analytical solution methods. However, 3D mechanisms, even simple mechanisms have no established method of analytical solution. But motion simulation can solve the problem easily in seconds, because it is designed to handle mechanisms of any and every complexity, both 2D and 3D. The mechanism may contain a large number of rigid links, springs, dampers, and contact pairs with virtually no penalty in solution time.

Firstly, the mechanism was modelled according to the parameters indicated and assembled with setting of all degrees of freedom, figure 5.

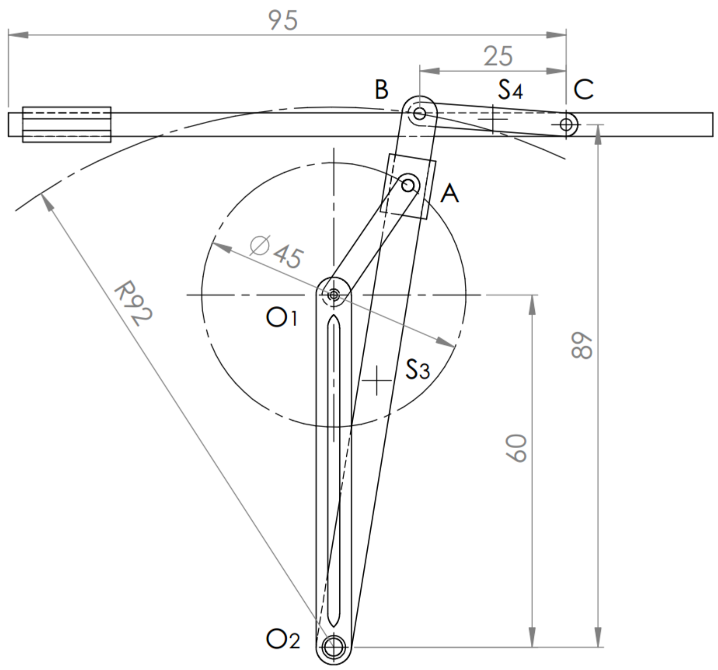


Figure 5. 3D model of studied rocker mechanism.

Then, from the SolidWorks program settings the Motion Analysis module was activated. And to the O_1A crank the rotational motion of 120 min^{-1} was assigned using the *RotaryMotor* function from the Motion Analysis settings bar. Also, the simulation time of 1 s was set (for 2 complete rotations) in order to plot clear and full diagrams, figure 6. After the analysis was generated, the next step is plotting the charts with the generated results. The chart plotting steps are shown in figure 7.

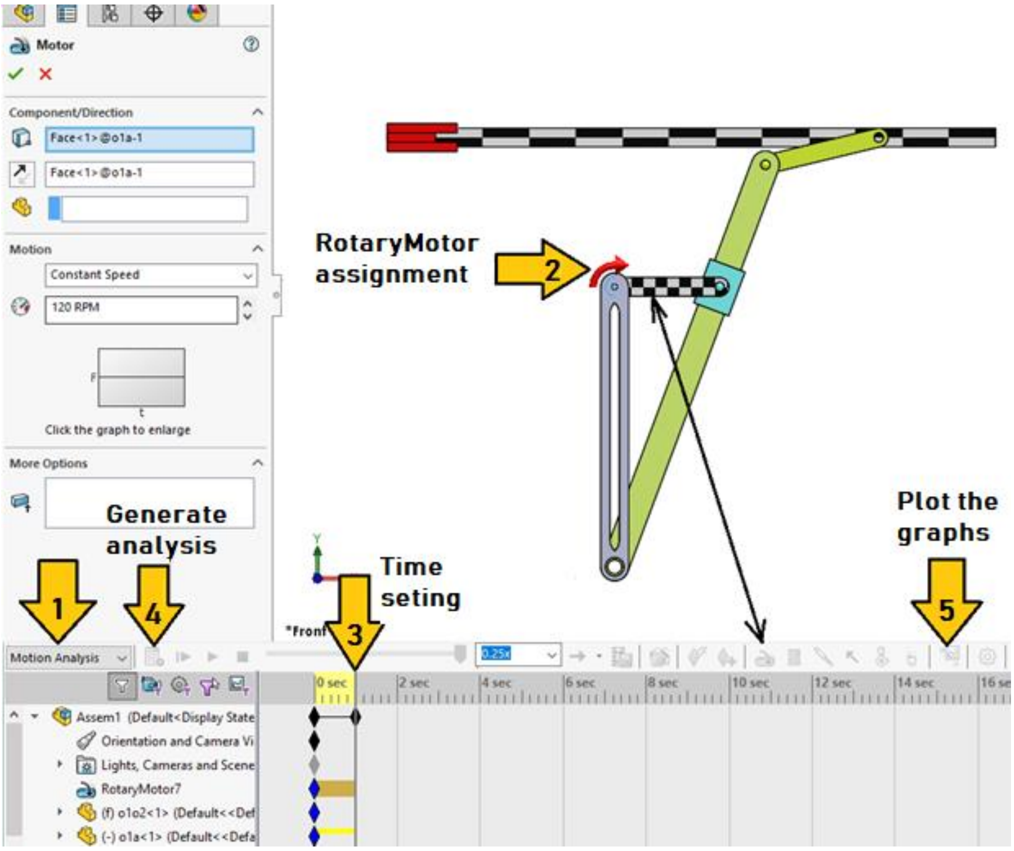


Figure 6. Rocker mechanism Motion Analysis settings.

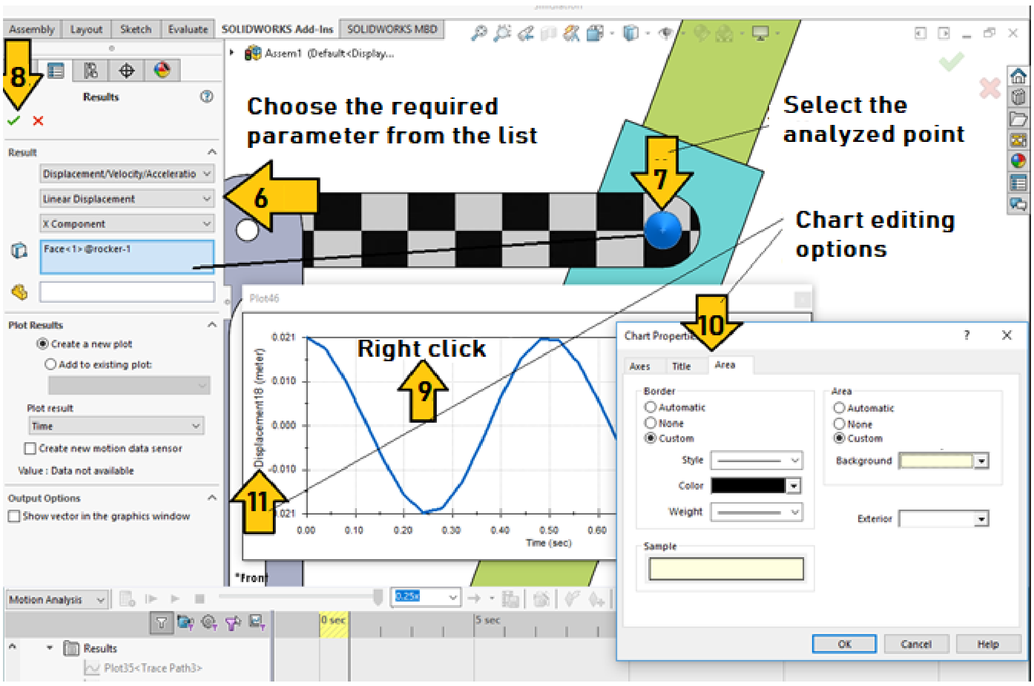
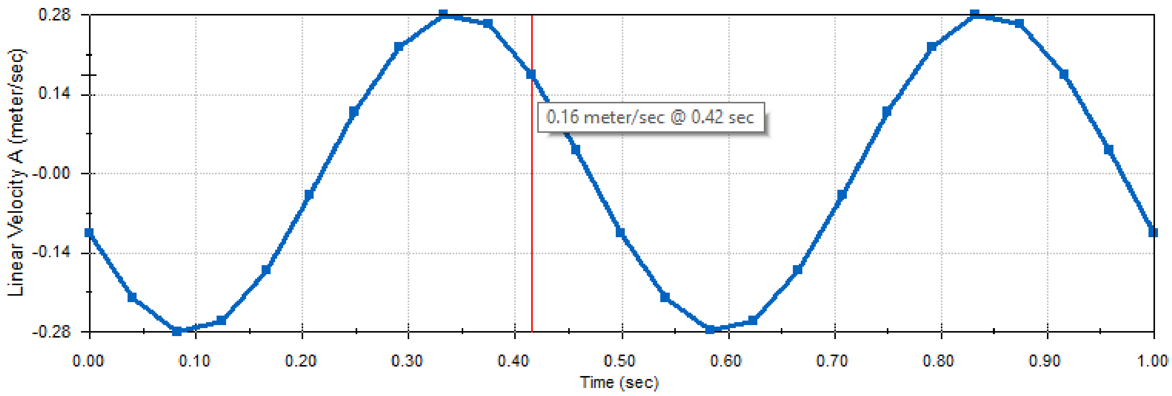
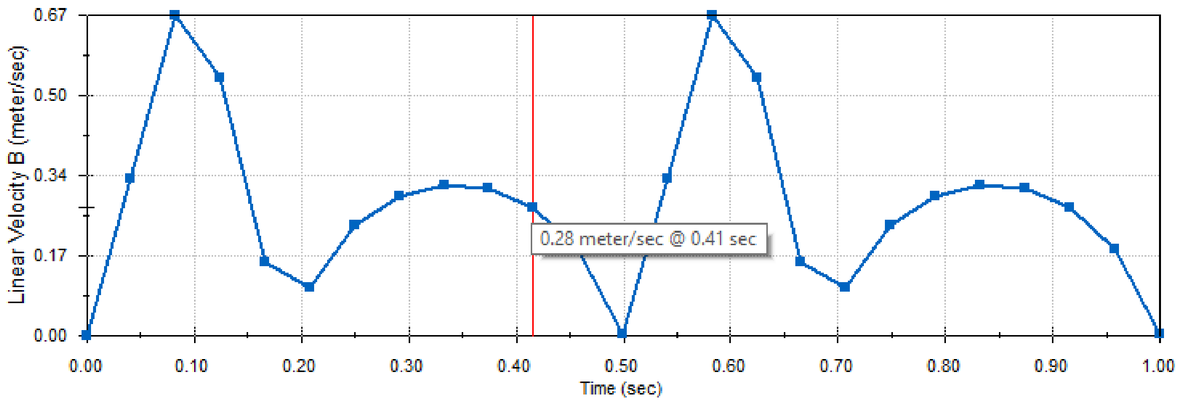


Figure 7. Chart plotting settings.

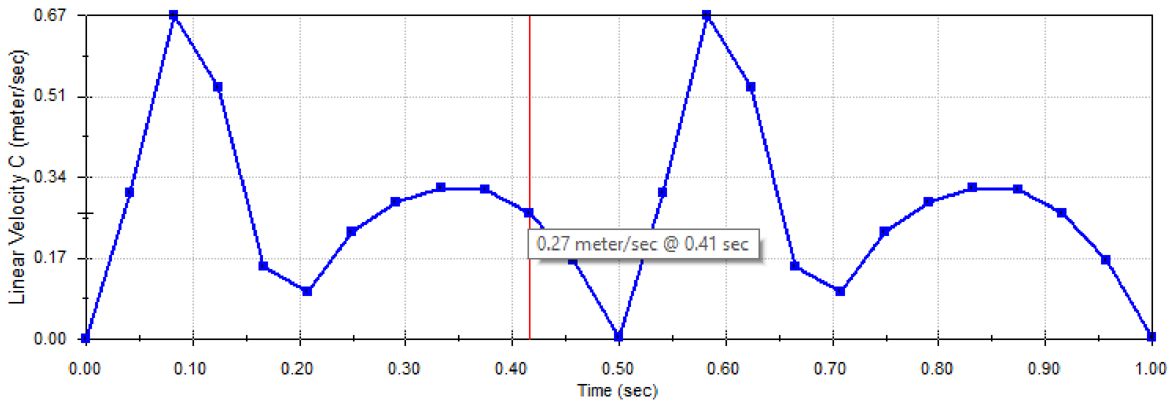
Thus, the diagrams of linear and angular velocities and accelerations were drawn for all kinematic coupling and elements analyzed by the graphic-analytical method (except Coriolis acceleration). The following figures show the graphs obtained from the kinematic analysis of the Rocker mechanism. In order to verify the accuracy of the results, in each chart the approximate value corresponding to the time when the O_1A crank is in the position of 45° and it was highlighted. As it can be seen, the obtained results validate the accuracy and high productivity of the computational method. Unfortunately, the unit of measurement of the angle could not be changed to radians. This is rather a shortcoming of this software version.



a)



b)



c)

Figure 8. Linear velocity distribution of the A, B, C coupling.

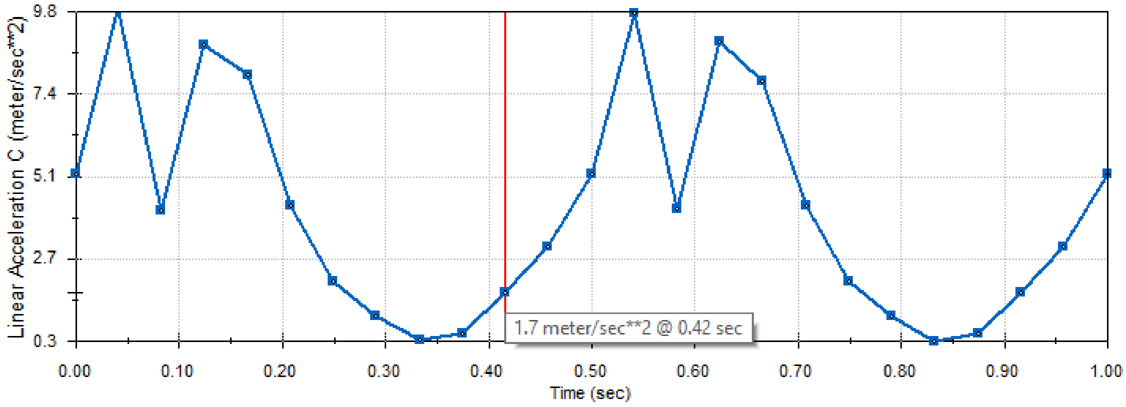


Figure 9. Linear acceleration distribution of the coupling C.

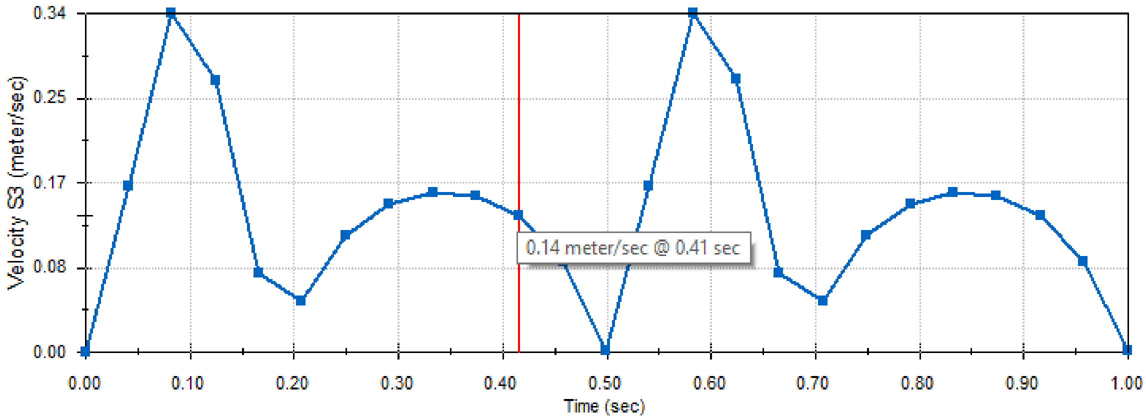


Figure 10. O_2B crank center of mass (S_3) velocity distribution.

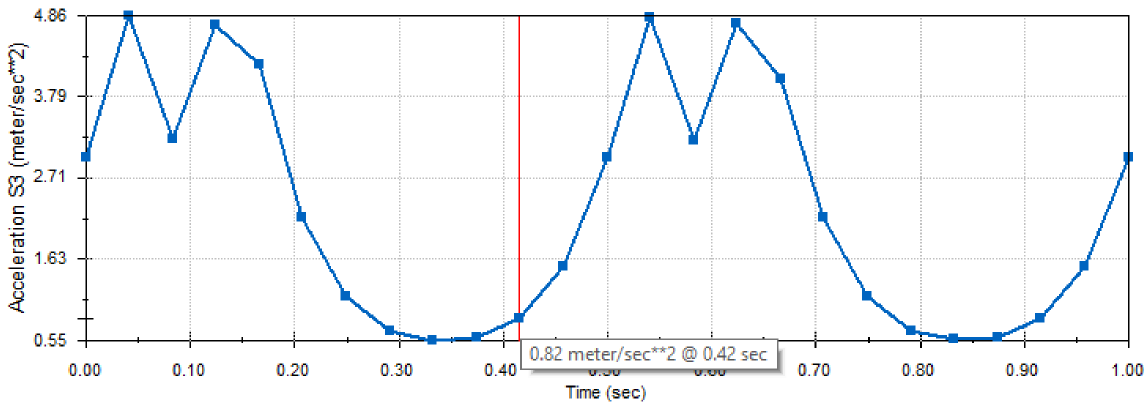


Figure 11. O_2B crank center of mass (S_3) acceleration distribution.

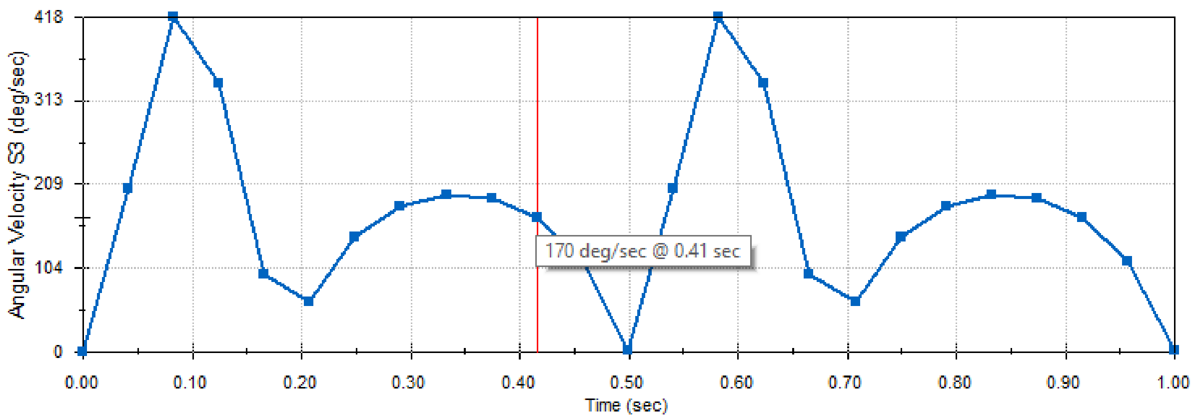


Figure 12. O_2B crank center of mass (S_3) angular velocity distribution.

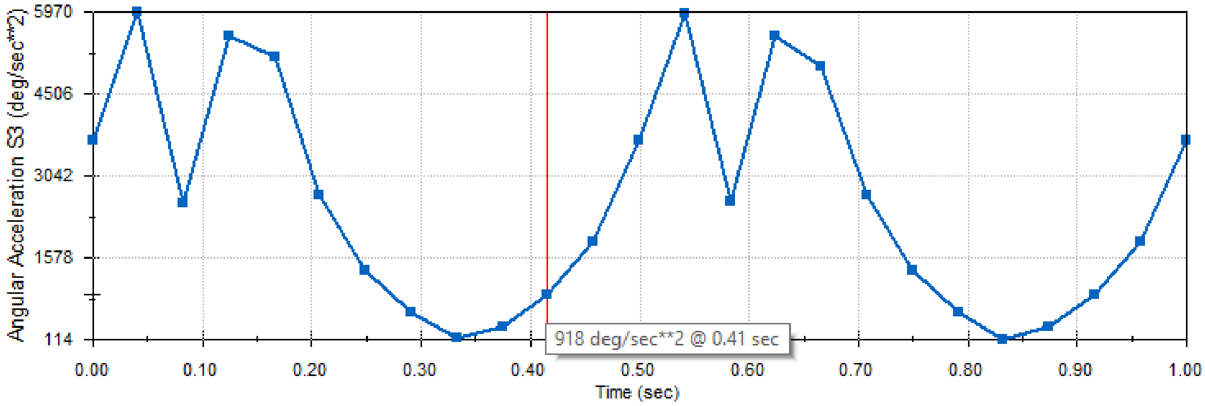


Figure 13. O_2B crank center of mass (S_3) angular acceleration distribution.

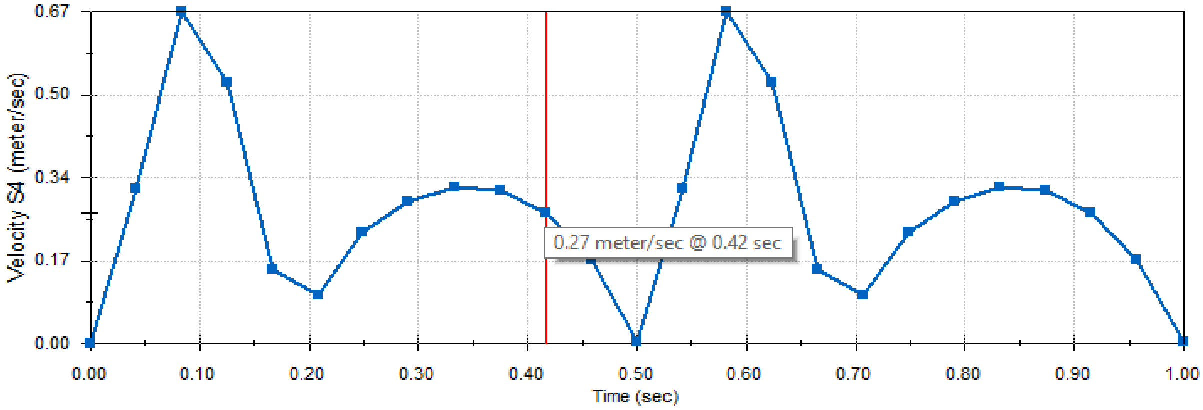


Figure 14. BC rod center of mass (S_4) velocity distribution.

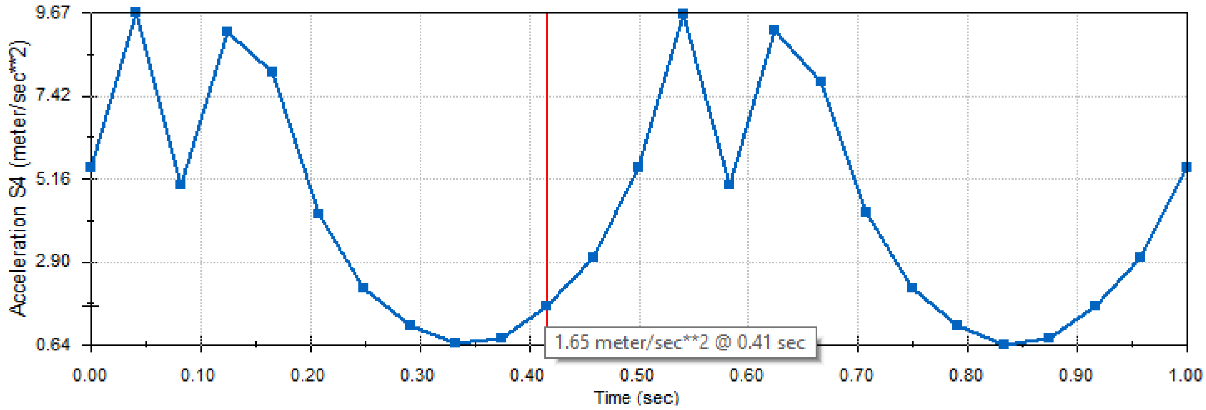


Figure 15. BC rod center of mass (S_4) acceleration distribution.

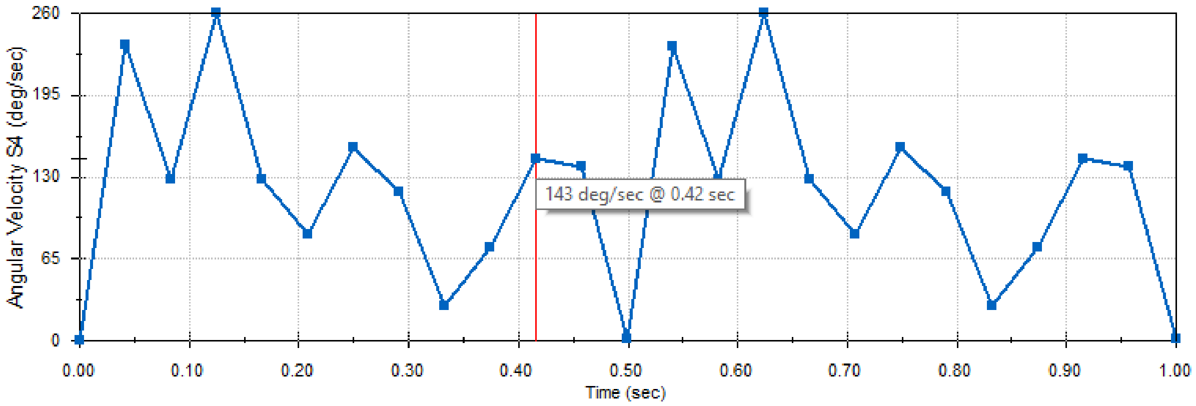


Figure 16. BC rod center of mass (S_4) angular velocity distribution.

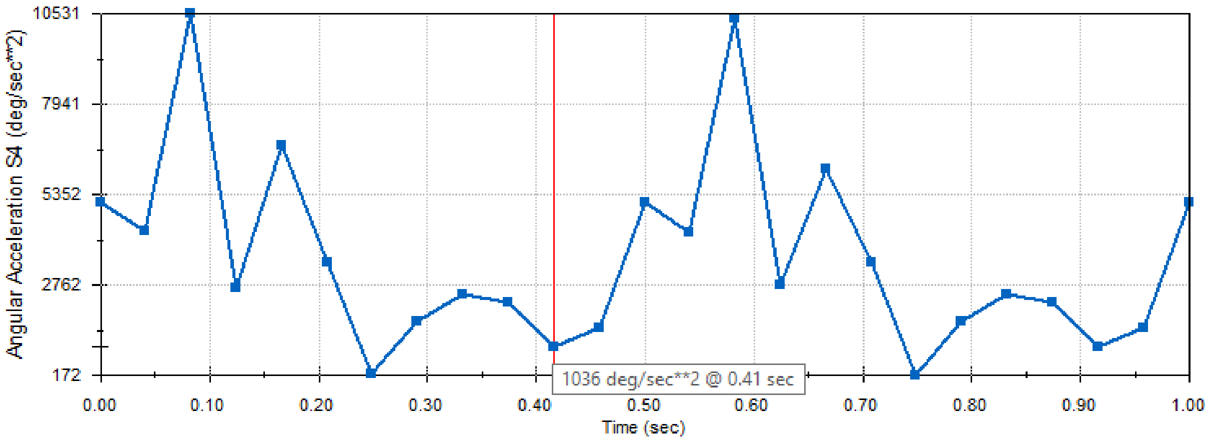


Figure 17. BC rod center of mass (S_4) angular acceleration distribution.

Conclusions

The kinematic analysis presented in this paper confirms the advantages of computer-aided engineering. As it can be seen, the obtained results validate the accuracy and high productivity of the computational method especially where complicated calculations with minimal errors are required.

Such kinematic analysis can be successfully implemented within specialized discipline teaching process altogether with the graphic-analytical method. This will help to increase the ability of the future engineers to use IT tools. The current development of engineering requires knowledge of IT tools. It facilitates learning of engineering objects and contributes to increase the labour productivity.

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